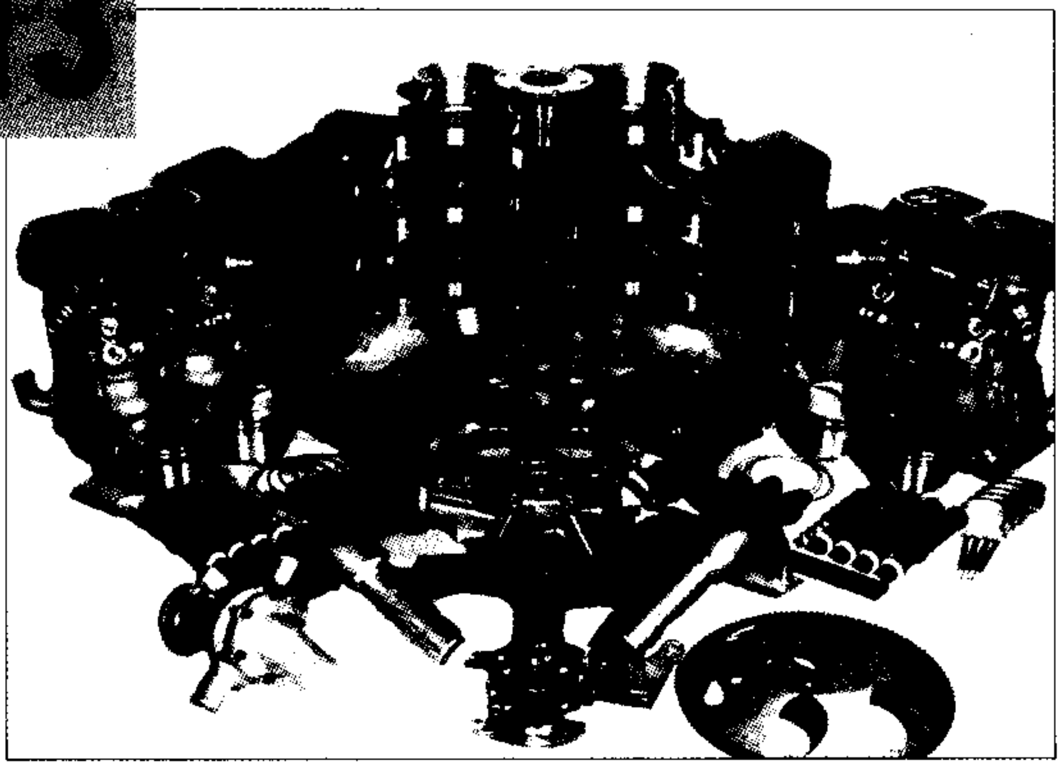
13



STATIC AND DYNAMIC FORCE ANALYSIS

13.1 INERTIA FORCES IN MECHANISMS

Forces in mechanisms arise from various sources, for example forces of gravity, forces of assembly, forces from applied loads, forces from energy transmission, frictional forces, inertia forces, spring forces, impact forces and forces due to change of temperature. All these forces must be considered in the final design of a machine for its successful operation.

13.2 STATIC FORCE ANALYSIS

In the analysis of static forces, the inertia forces are not taken into account. Often the gravity forces are also small and are neglected as compared to other forces.

13.2.1 Slider Crank Mechanism

Figure 13.1 (a) shows a slider–crank mechanism. A force P is applied to the piston due to gas pressure. As a result of it, to maintain equilibrium, a torque T_2 must be exerted on crank 2 by the shaft at O_2. The procedure followed for the static force analysis is to draw the free body diagram for the various links showing all the external forces and moments acting on the link. If the number of unknowns are not more than three then equations of equilibrium are used to solve the problem. But if the number of unknowns are more than three then additional information is obtained from the equilibrium of the other links.

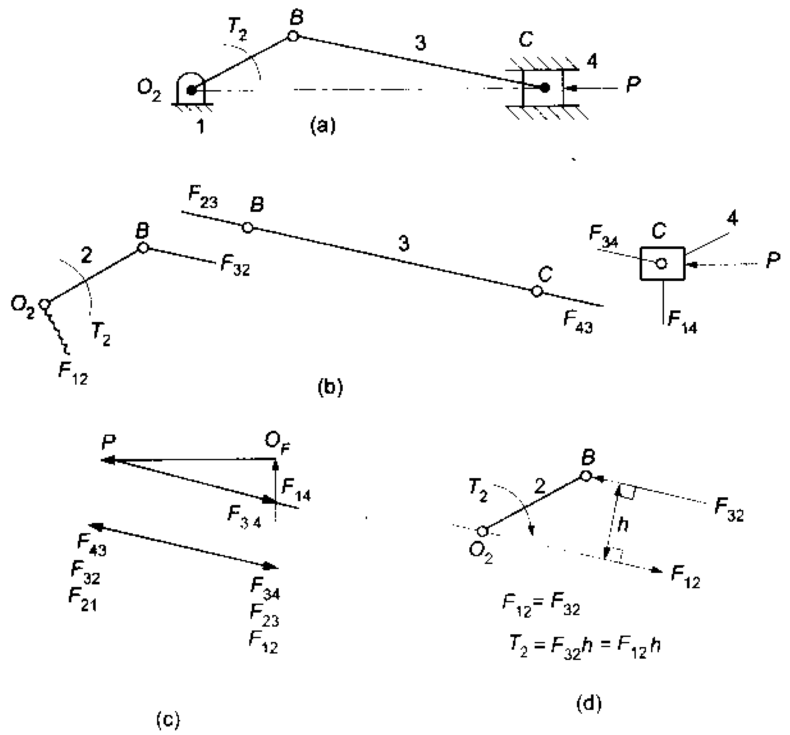


Fig.13.1 Static force analysis of slider–crank mechanism

Figure 13.1(b) shows the free body diagram for all the links of the mechanism. At a pin connection there can be no moments. When a body is in equilibrium under the action of two forces only, such a body is called a *two-force member*. F_{ij} is the force which the body i exerts on the body j. We shall use a solid line without an arrowhead to indicate that the direction of a force is known but its magnitude is unknown. Link 3 has two forces and link 4 has three forces acting on it. Force P is known in magnitude and direction. The two unknown forces for link 4 are F_{34} and F_{14} in magnitude only. Link 2 has four unknowns: force F_{32} known in direction only, force F_{12} unknown in magnitude and direction and the unknown moment T_2 exerted on crank 2 by the shaft. A wavy line placed at O_2 indicates that we do not know the magnitude or direction of the force F_{12}, which acts through that point.

Link 4, which has only two unknowns, is analyzed first. The two unknown magnitudes can be found by laying out a force polygon as shown in Fig.13.1(c). From Fig.13.1(d), we note that F_{12} must be equal and opposite to F_{32} to balance forces on link 2. However, the two equal, opposite and parallel forces produce a couple which can be balanced by another couple only. The balancing couple T_2 is equal to $F_{32}\,h$, where h is the perpendicular distance between F_{32} and F_{12}. It is clockwise and is the torque which the shaft exerts on the crank 2.

13.2.2 Four-bar Mechanism

Consider a four-bar mechanism as shown in Fig.13.2(a), subjected to two forces, P and Q. A moment T_2 must be applied to link 2 to maintain equilibrium. The free body diagram of the various links is shown in

Fig.13.2(b). The unknowns for the various links are: five for link 2, four for link 3 and four for link 4. Therefore, these links cannot be solved by the equilibrium equations. If we consider links 3 and 4 together then there are six unknowns, because $F_{ij} = F_{ji}$. Since there are six equations of equilibrium, three for each link, we can obtain a solution. The forces on links 3 and 4 are shown in Fig.13.2(c). Force F_{34} is broken into components F_{34}^n and F_{34}^t, which are parallel and perpendicular respectively, to O_4C. The magnitude of F_{34}^t is found by taking moments about O_4, that is,

$$F_{34}^t = \frac{Pa}{O_4C}$$

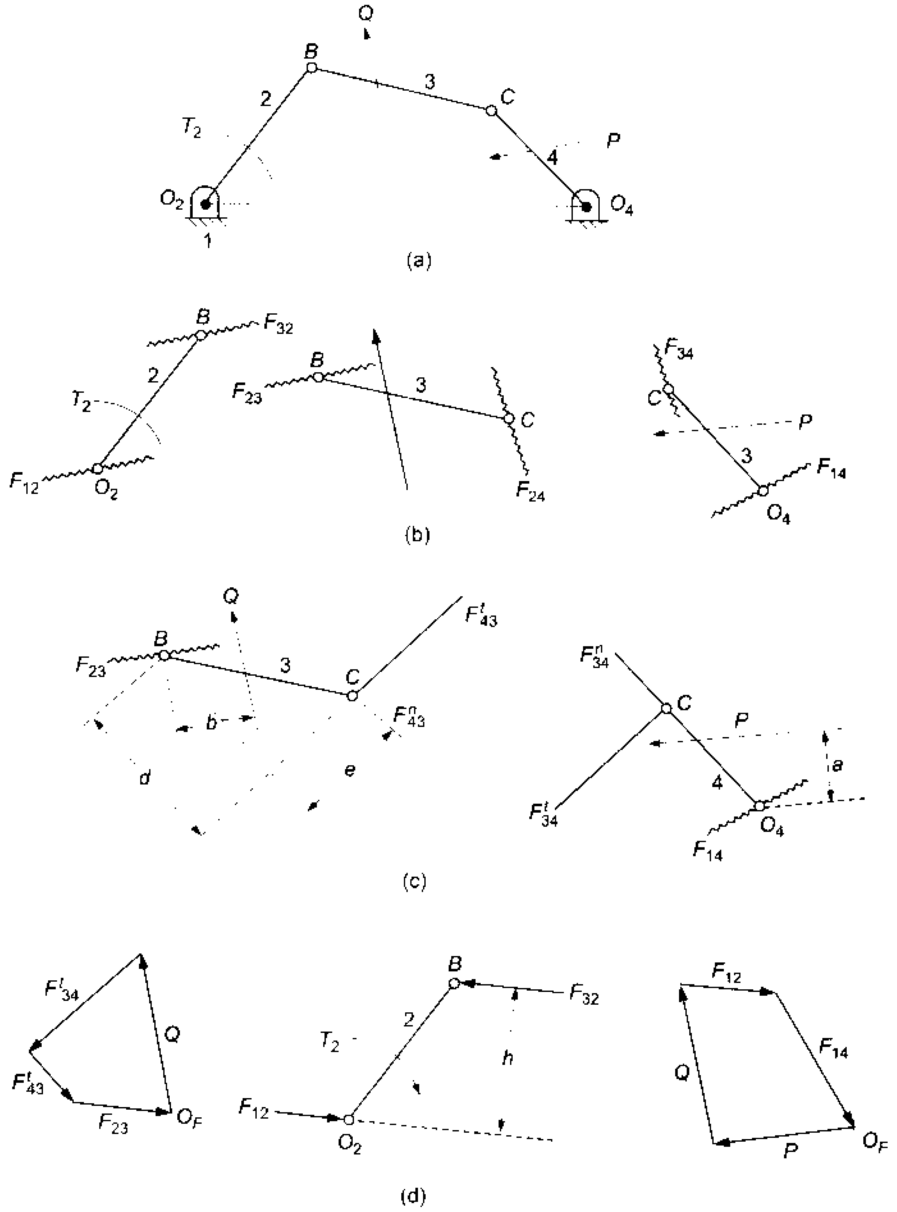


Fig.13.2 Static force analysis of four-bar chain

On link 3 the reactions at C are equal and opposite to those at C on link 4. The magnitude of $F_{43}^t = F_{34}^t$. On link 3 , there are three unknowns: magnitude and direction of F_{23} and magnitude of F_{43}^n. The magnitude of F_{43}^n can be found by taking moments about point B.

$$Qb - F_{43}^t d + F_{43}^n e = 0$$

or

$$F_{43}^n = \frac{F_{43}^t d - Qb}{e}$$

Next we draw the force polygon for link 3, as shown in Fig.13.2(d), to obtain the magnitude and direction of F_{23}.

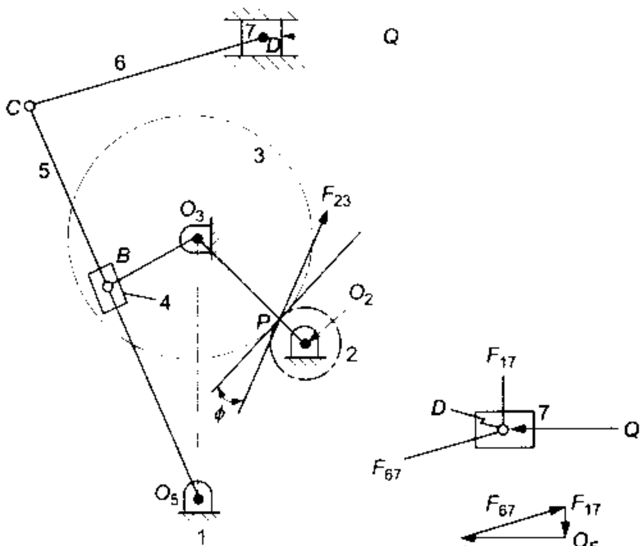
In Fig.13.2(e), $F_{32} = F_{23}$. Then $F_{12} = F_{32}$. Taking moments about O_2, we obtain T_2, the torque which the shaft at O_2 exerts on link 2.

$$T_2 = F_{32}h$$

Force F_{14} is obtained from the force polygon for bodies 2, 3 and 4, taken as a whole system as shown in Fig.13.2(f).

13.2.3 Shaper Mechanism

The shaper mechanism is shown in Fig.13.3(a). We begin by considering link 7 as a free body, as shown in Fig.13.3(b). The direction of F_{67} and F_{17} are known and their magnitudes can be found from a force polygon as shown. In Fig.13.3(c), the free body diagram for link 6 is drawn. $F_{76} = F_{67}$, and because 6 is a two-force member, therefore $F_{56} = F_{76}$.



(a) (b)

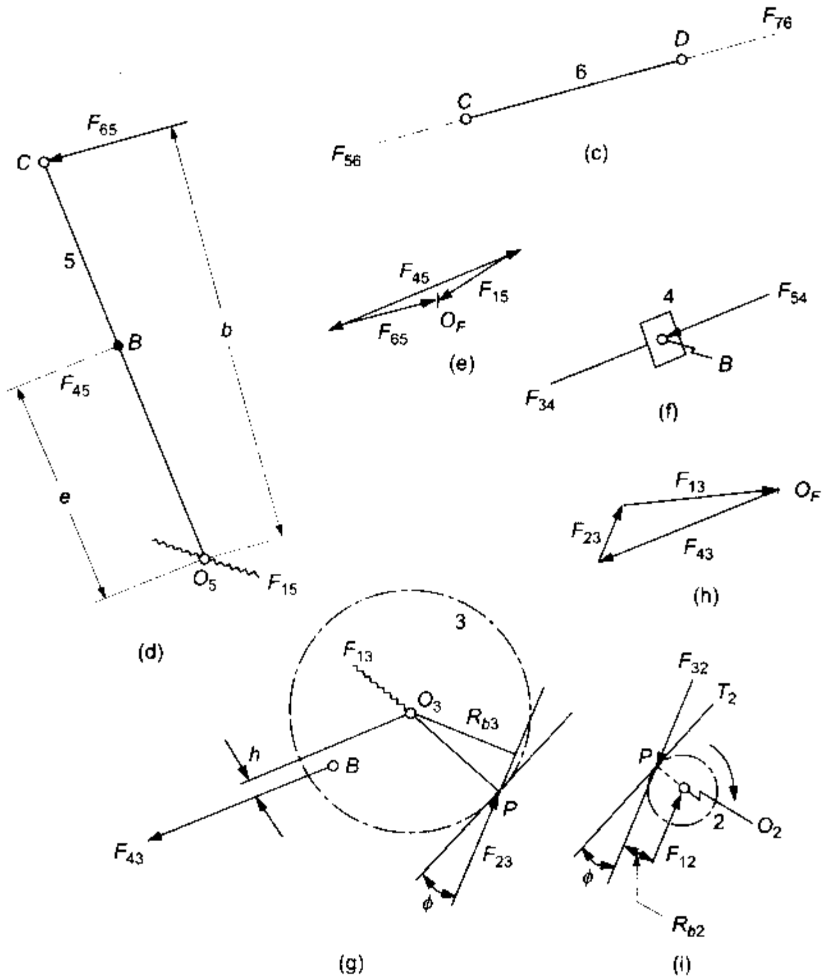


Fig.13.3 Static force analysis of shaper mechanism

The free body diagram of link 5 is shown in Fig.13.3(d), where $F_{65} = F_{56}$. Force F_{45} is directed perpendicular to link 5 but its magnitude is unknown. F_{15} is unknown in magnitude and direction. Taking moments about O_5, we have

$$F_{65}b = F_{45}e$$

or

$$F_{45} = \frac{F_{65}\,b}{e}$$

The magnitude and direction of F_{15} can be determined from force polygon shown in Fig.13.3(e). The free body diagram for slider 4 is shown in Fig.13.3(f), where $F_{54} = F_{45}$. Also $F_{34} = F_{54}$.

The free body diagram for link 3 is shown in Fig.13.3(g), where $F_{45} = F_{34}$ and F_{13} is unknown in magnitude and direction. The values of moment arm h and the radius R_{b3} of the base circle can be measured, where ϕ is the pressure angle of the gear. By taking moments about O_3, the magnitude of F_{23} can be calculated. Next the magnitude and direction of F_{13} can be found from a force polygon as shown in Fig. 13.3(h). Finally, from Fig.13.3(i), $F_{32} = F_{23}$ and $F_{12} = F_{32}$. The torque exerted by the pinion shaft on the pinion, $T_2 = F_{32} \cdot R_{b2}$ and is clockwise.

13.3 INERTIAL FORCE ANALYSIS

Consider a body whose centre of mass is G, its linear acceleration a_G and angular acceleration α, as shown in Fig.13.4(a). Let a force $F = ma_G$ be applied at G, from left to right upwards. This force can be replaced by another force F acting at a distance h [Fig.13.4(b)] together with a torque $T = I\alpha$, where I is the moment of inertia of the body about an axis passing through G and perpendicular to the plane of rotation.

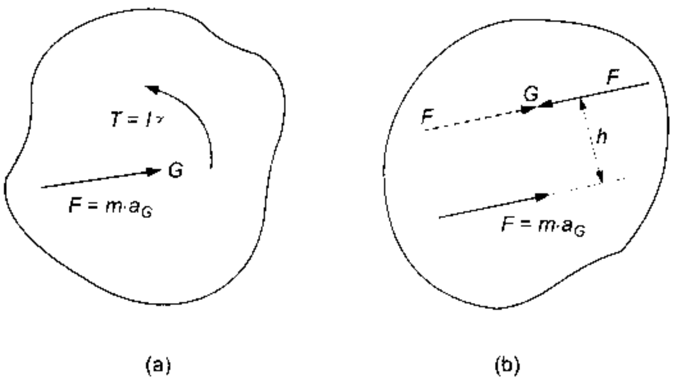


Fig.13.4 Inertia force in a four-bar mechanism

Hence $$Fh = T = I\alpha$$

or $$h = \frac{I\alpha}{F} \qquad (13.1)$$

In general F is the resultant force and T is the resultant torque acting on the body. The inertia force is defined as the reversed resultant force and the inertia torque is defined as the reversed resultant torque. If the inertia force is considered along with the resultant force, then the body would be in equilibrium. Similarly, if the inertia torque is considered along with the resultant torque, then the angular acceleration of the body would be zero. Hence,

Hence, $$F - ma_G = 0 \qquad (13.2a)$$

and $$T - I\alpha = 0 \qquad (13.2b)$$

This is known as the *D'Alembert principle* and aids in the solution of problems in dynamics by permitting them to be solved as problems in statics.

13.3.1 Inertia Force in a Four-bar Mechanism

Consider a four-bar mechanism shown in Fig.13.5(a), where the magnitude of ω_2 is assumed known and constant. Points G_2, G_3 and G_4 denote the centres of mass of the links 2, 3, and 4 respectively. We are interested in determining the torque which the shaft at O_2 must exert on crank 2 to give the desired motion.

To determine the linear acceleration of the points G_2, G_3 and G_4, we construct the acceleration polygon. From the magnitude and sense of the tangential components of acceleration, the magnitude and sense of α_3 and α_4 can be determined.

Link 2 is shown in Fig.13.5(c), where a_{G2} is the acceleration of the centre of mass G_2. The resultant force $F_2 = m_2 a_{G2}$,where m_2 is the mass of the link 2, has the same sense and line of action as a_{G2}. The inertia force $f_2 = -F_2$.

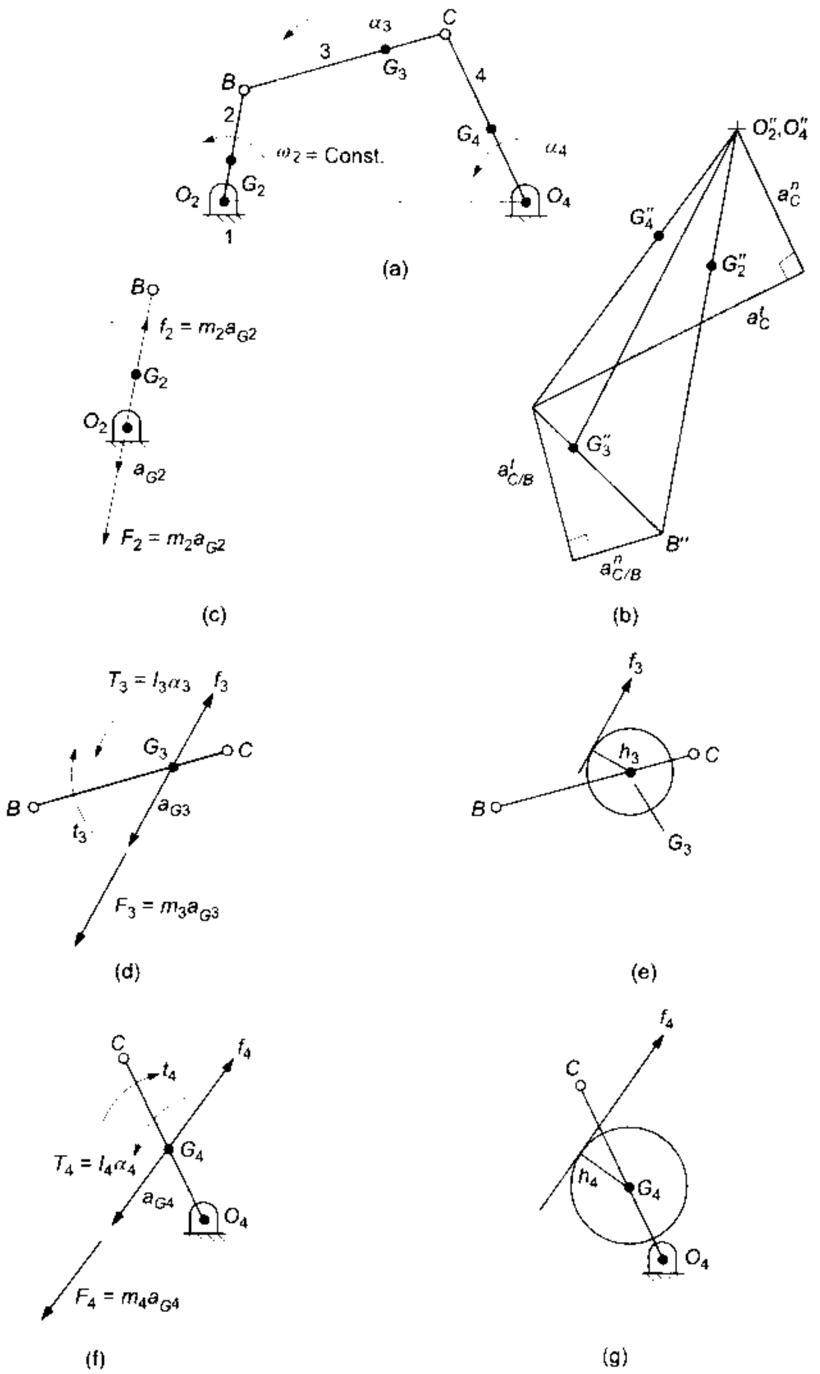


Fig.13.5 Inertia force in a four-bar mechanism

Link 3 is shown in Fig.13.5(d) with the acceleration of the centre of mass G_3 indicated as a_{G3}. The resultant force $F_3 = m_3 a_{G3}$, where m_3 is the mass of the link 3, has the same sense and line of action as a_{G3}.

$f_3 = -F_3$ is the inertia force. In order to produce α_3, there must be a resultant torque $T_3 = I_3\alpha_3$ having the same sense as α_3. Inertia torque $t_3 = -T_3$. Link 3 is again shown in Fig.13.5(e), where the inertia force f_3 and inertia torque t_3 have been replaced by a single force f_3. The direction and sense of f_3 is the same as in Fig.13.5(d), but the line of action is displaced from G_3 by an amount h_3, such that

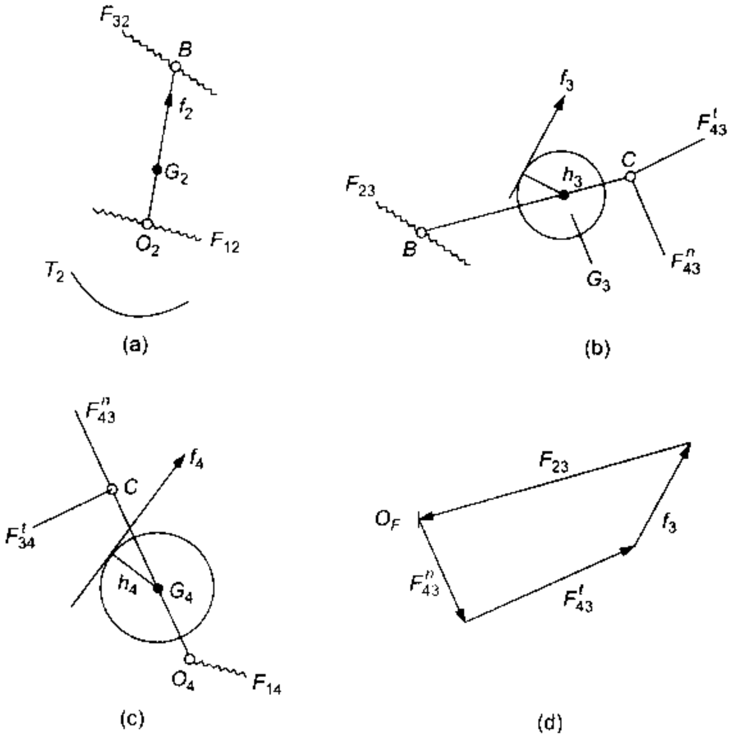
$$f_3 h_3 = t_3 \quad \text{or} \quad h_3 = \frac{t_3}{f_3} = \frac{I_3\alpha_3}{m_3 a_{G3}}$$

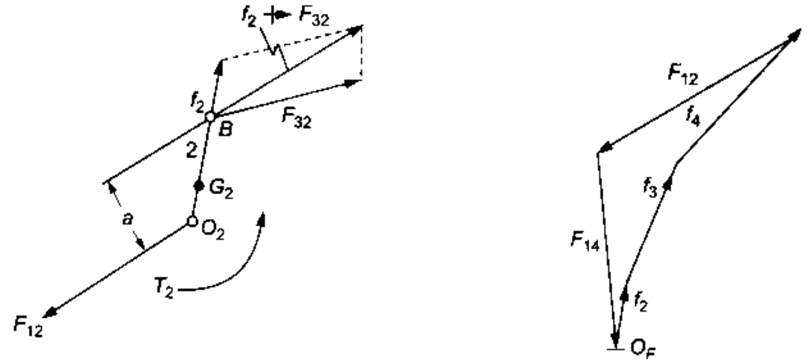
In Fig.13.5(e), f_3 can be located by drawing a circle of radius h_3 with its centre at G_3. Now, f_3 is drawn tangent to the left side of the circle rather than the right side because f_3 must produce a torque about G_3 in the same sense as t_3.

Link 4 is shown in Fig.13.5(f) where $f_4 = -F_4$ and $T_4 = I_4\alpha_4$. Inertia torque $t_4 = -T_4$. Link 4 appears again in Fig.13.5(g), where the inertia force f_4 and inertia torque t_4 have been replaced by a single force f_4. Since $f_4 h_4$ must equal t_4,

$$h_4 = \frac{t_4}{f_4} = \frac{I_4\alpha_4}{m_4 a_{G4}}$$

To find the forces at each pin connection and the torque which the shaft exerts on crank 2, we draw the free body diagrams of links 2, 3, and 4 as shown in Fig.13.6(a) to (c). For the known inertia forces in each link, the forces in each pin can be determined by using the equilibrium equations. Starting with link 4, we take the moments about point O_4 and determine F_{34}^t. Then on link 3, $F_{43}^t = -F_{34}^t$. For equilibrium of link 3, the sum of the moments about B equals zero. This determines F_{43}^n. The force polygon for link 3 is shown in Fig.13.6(d) to determine F_{23}.





(e) (f)

Fig.13.6 Force polygons

Link 2 appears in Fig.13.6(e). Here $F_{32} = -F_{23}$. Then $F_{12} = -(f_2 + F_{32})$. Taking moments about O_2, we obtain T_2, as

$$T_2 = (f_2 + F_{32})a$$

Force F_{14} is obtained from the force polygon for bodies 2, 3, and 4 taken as a whole system as shown in Fig.13.6(f).

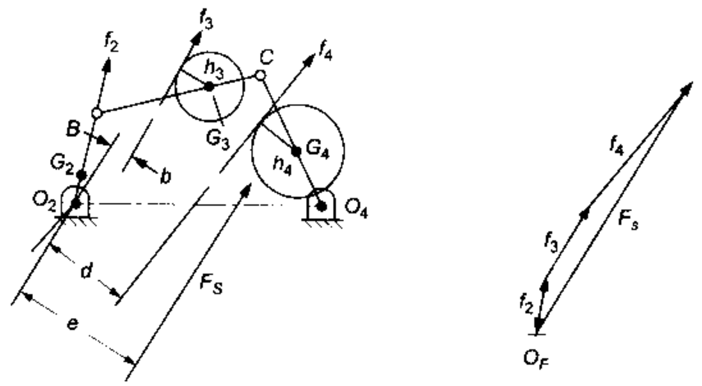
Shaking force It is defined as the resultant of all the forces acting on the frame of a mechanism due to inertia forces only.

The inertia forces on a four-bar mechanism are shown in Fig.13.7(a). The force polygon is shown in Fig.13.7(b). Taking moments about point O_2, we get

$$F_s e = f_3 b + f_4 d$$

or

$$e = \frac{f_3 b + f_4 d}{F_s}$$



(a) (b)

Fig.13.7 Force polygons

13.4 COMBINED STATIC AND DYNAMIC FORCE ANALYSIS

13.4.1 Slider–crank Mechanism

The slider–crank mechanism is shown in Fig.13.8(a). Let P be the force on the piston due to gas pressure and ω_2 the angular velocity of link 2, be known. Points G_2, G_3 and G_4 are the centres of mass of links 2, 3, and 4. We are interested to find the torque T_2 which the crank 2 exerts on the crankshaft and the shaking force.

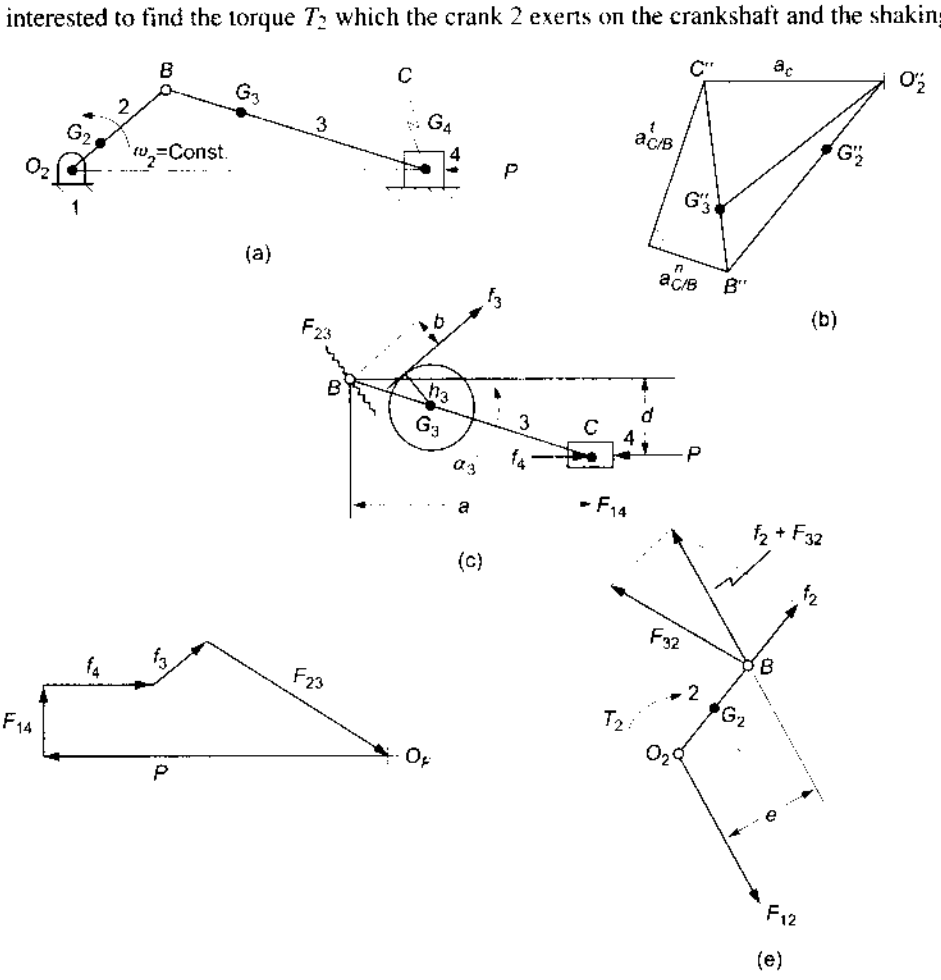


Fig.13.8 Static and inertia force analysis of slider–crank mechanism

The velocity and acceleration polygons are constructed first, as shown in Fig.13.8(b). Links 3 and 4 combined as a free body are shown in Fig.13.8(c). The inertia force f_3, its moment about G_3 and f_4 are determined as explained in Section 13.3. The unknowns are the magnitudes of F_{23} and F_{14}. By taking moments about B, we have

$$F_{14}a + f_3b + f_4d - Pd = 0$$

or

$$F_{14} = \frac{Pd - f_3b - f_4d}{a}$$

Force F_{23} can then be found by a summation of forces on bodies 3 and 4 together as a free body. The force polygon is shown in Fig.13.8(d).

The free body diagram for link 2 is shown in Fig.13.8(e), where

$$F_{12} = -(f_2 + F_{32})$$

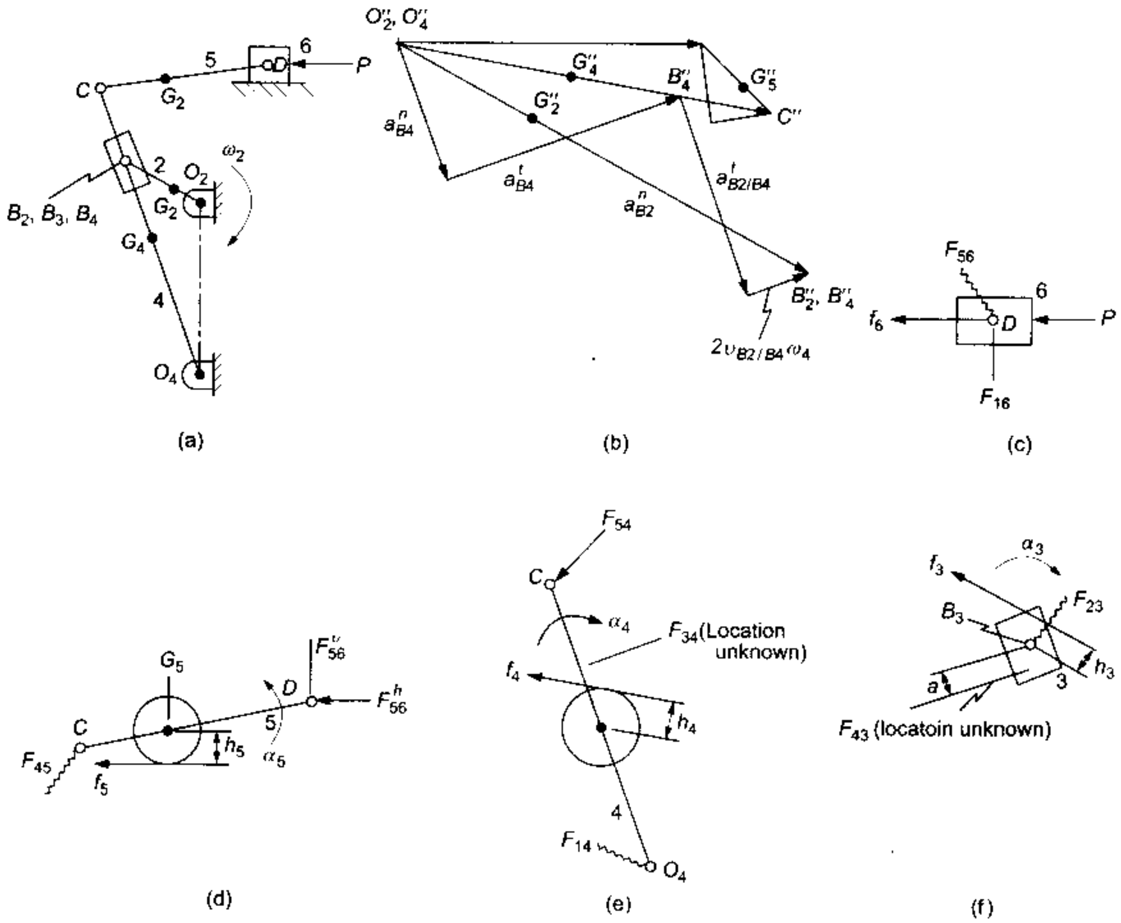
The torque exerted by the shaft on the crank 2 at O_2 is,

$$T_2 = (f_2 + F_{32})e$$

The torque exerted by the crank on the crankshaft is equal to T_2 but opposite in sense.

13.4.2 Shaper Mechanism

Consider the shaper mechanism shown in Fig.13.9(a), where the link 2 rotates with constant velocity ω_2 and the force P is known on the slider 6. We are interested to determine the forces at all the points and the torque T_2 exerted by the shaft to drive the crank. The acceleration diagram is shown in Fig.13.9(b).



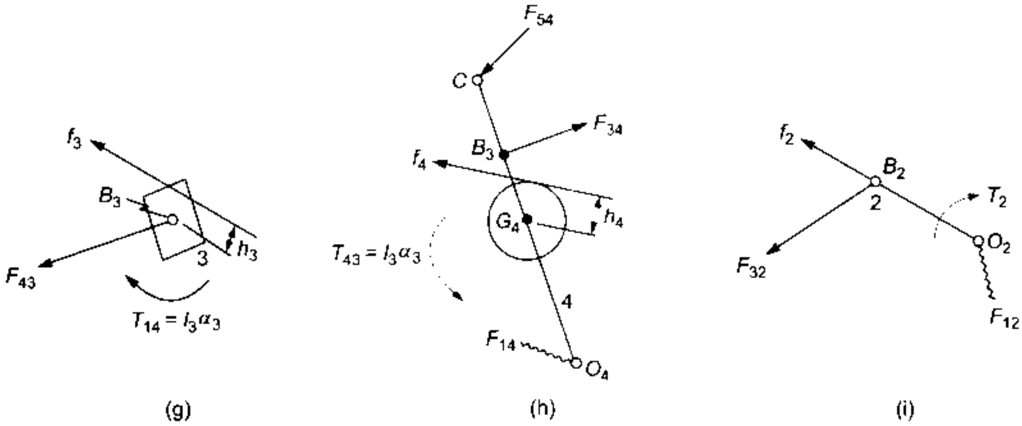


Fig.13.9 Static and inertia force analysis of shaper mechanism

The force analysis is started with link 6, shown in Fig.13.9(c). The unknowns are: magnitude of F_{16}, magnitude and direction of F_{56}. The horizontal component of F_{56} is F_{56}^h and its magnitude can be found from the summation of horizontal forces on link 6.

In Fig.13.9(d), $F_{65}^h = -F_{56}^h$. The magnitude of F_{65}^v can be found by summation of moments about C. Then from a force polygon for link 5, the magnitude and direction of F_{45} are found. Next, in Fig.13.9(e), $F_{54} = -F_{45}$ is known. There are four unknowns in Fig.13.9(e): the magnitude and direction of F_{14} and the magnitude and location of F_{34}. For link 3 shown in Fig.13.9(f), there are also four unknowns: F_{23} in magnitude and direction and F_{43} in magnitude, which is perpendicular to link 4. However, for the combination of links 3 and 4, there are six unknowns that can be analyzed in combination. From the free body of link 3 we see that F_{23} causes no torque about the centre of mass B_3, and thus F_{43} must be of such a magnitude as to balance the forces, and its line of action must be displaced from B_3 enough to balance the inertia torque. F_{43} can then be resolved into a force passing through B_3 and a torque about B_3 sufficient to balance the inertia torque. This is shown in Fig.13.9(g). The equal and opposite force and torque on link 4 as shown in Fig.13.9(h) makes the free body of link 4 with three unknowns. The magnitude of F_{34} can be found by setting the sum of the moments about O_4 equal to zero. F_{14} can then be found from a force polygon for link 4.

We replaced F_{43} in Fig.13.9(f) with the force F_{43} and T_{43}, which are shown in Fig.13.9(g). Thus in Fig.13.9(f),

$$F_{43}a = T_{43} = I_3\alpha_3$$

or

$$a = \frac{I_3\alpha_3}{F_{43}}$$

F_{23} can now be determined from a force polygon for link 3. The free body diagram of link 2 is shown in Fig.13.9(i) and $F_{32} = -F_{23}$. F_{12} can be determined from a force polygon on link 2. Finally, by summing moments about O_2, the torque T_2 can be determined.

Example 13.1

The links 3 and 4 of a four-bar mechanism are subjected to forces of 100 $N\angle 60°$ and 50 $N\angle 45°$. The dimensions of various links are:

$O_2O_4 = 800$ mm, $O_2B = 500$ mm, $BC = 450$ mm, $O_4C = 300$ mm, $BD = 200$ mm, $O_4E = 150$ mm. Calculate the shaft torque T_2 on the link 2 for static equilibrium of the mechanism. Also find the forces in the joints.

Solution

The mechanism has been drawn in Fig.13.10(a).

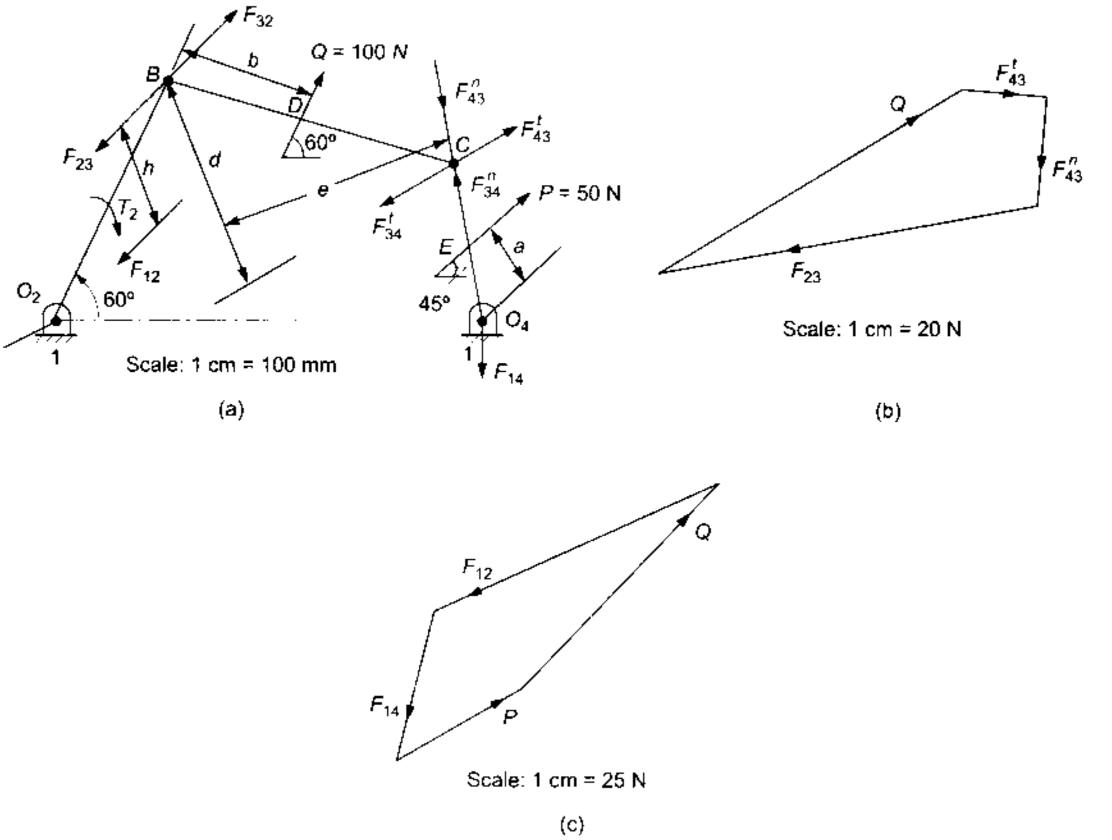


Fig.13.10 Four-bar mechanism

Let F_{34}^t and F_{34}^n be the forces at joint C on link 4, perpendicular and parallel to the link O_4C. Draw a line parallel to force $P = 50\ \text{N}\angle 60°$. The perpendicular distance between these two lines is $a = 140$ mm.

Taking moments about O_4, we get

$$F_{34}^t = \frac{Pa}{O_4C} = \frac{50 \times 140}{300} = 23.33\ \text{N}$$

$$F_{43}^t = -F_{34}^t = 23.33\ \text{N}$$

Measure distances $b = 200$ mm, $d = 320$ mm, $e = 310$ mm from joint B of forces Q, F_{43}^t and F_{43}^n respectively. Taking moments about joint B, we get

$$Qb + F_{43}^t \times d - F_{43}^n \times e = 0$$

$$F_{43}^n = \frac{100 \times 200 + 23.33 \times 320}{310} = 30.53\ \text{N}$$

$$F_{34}^n = -F_{43}^n$$

Knowing forces Q, F_{43}^t and F_{43}^n, draw the force polygon to obtain F_{23} from Fig.13.10(b).

By measurement,

$$F_{23} = 108 \text{ N}$$
$$F_{32} = -F_{23}$$
$$F_{12} = F_{32} = 108 \text{ N}$$
$$T_2 = F_{32} \times h = 108 \times 180 = 19440 \text{ Nmm (cw)}$$

Torque exerted by the crankshaft on link 2 = 19440 N mm (ccw)

Now complete the force polygon for all the forces acting on the mechanism, as shown in Fig.13.10(c). F_{14} = 52.5 N.

Example 13.2

Link O_4C of a four-bar chain is subjected to a torque T_4 = 1 Nm ccw. The link BC is subjected to a force $Q = 45\angle 90°$ downwards. Determine the torque T_2 on link O_2B and the reactions at O_2 and O_4. The lengths of the various links are O_2O_4 = 90 mm, O_2B = 50 mm, BC = 55 mm, O_4C = 30 mm and $BD = BC$ = 27.5 m.

■ Solution

The mechanism has been drawn in Fig.13.11(a). Taking moments about O_4, we have

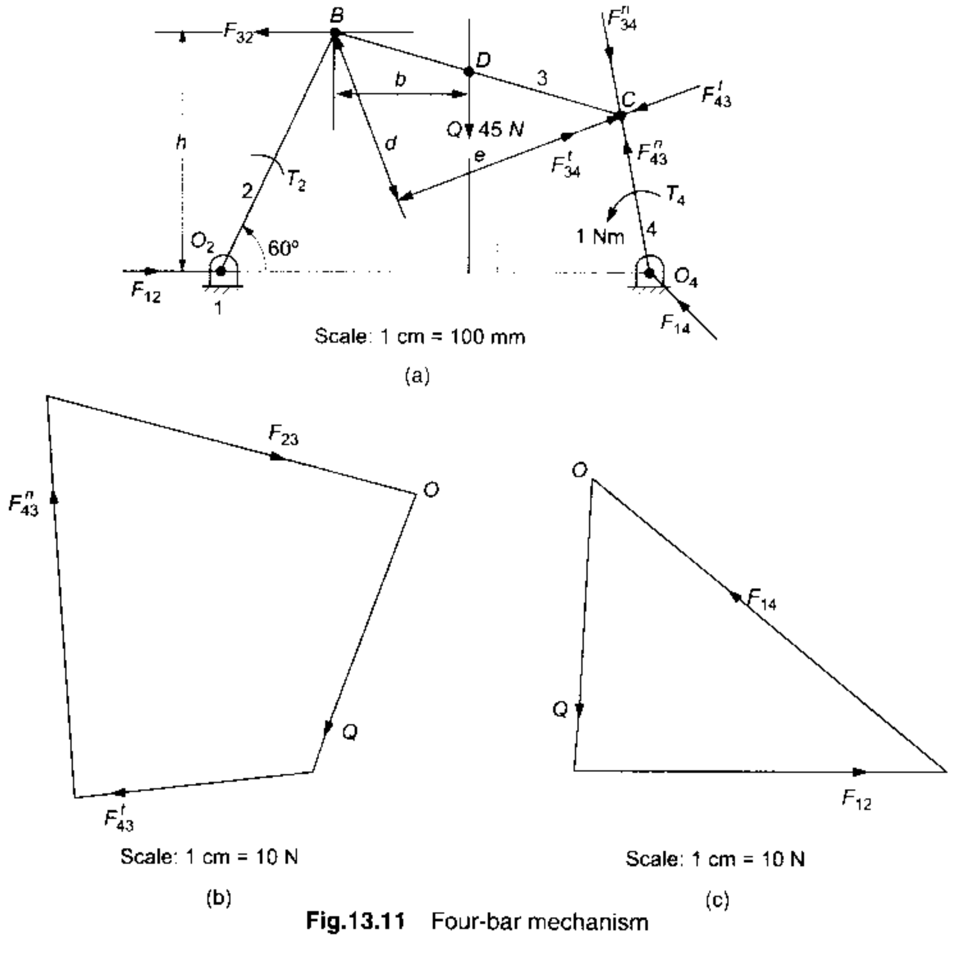


Fig.13.11 Four-bar mechanism

$$F_{34}^t \times O_4C = T_4 = 1$$

$$F_{34}^t = \frac{1}{0.03} = 33.3 \text{ N}$$

$$F_{34}^t = -F_{43}^t$$

Measure perpendicular distances $b = 26$ mm, $d = 36$ mm and $e = 40$ mm, as shown in Fig.13.11(a). Taking moments about joint *B*, we have

$$Qb - F_{43}^n \times e = F_{43}^t \times d = 0$$

$$F_{43}^n = \frac{45 \times 26 + 33.3 \times 36}{40} = 59.22 \text{ N}$$

$$F_{34}^n = -F_{43}^n$$

Draw the force polygon for link *BC*, as shown in Fig.13.11(b).

$$F_{23} = 53; \quad \text{N} F_{32} = -F_{23}$$

Also $F_{12} = F_{32}$.

Now draw the force polygon for Q, F_{12} and F_{14}, as shown in Fig.13.11(c). $F_{14} = 63$ N and $h = 42$ mm. Then

$$T_2 = F_{32}h = 53 \times 0.042 = 2.226 \text{ Nm (ccw)}$$

Torque on link $O_2B = 2.226$ Nm (cw)

Example 13.3

For the four-bar chain shown in Fig.13.12(a), T_3 on link *BC* is 30 N m clockwise and T_4 on *CD* is 20 Nm counter-clockwise. Find the torque exerted by crankshaft on *AB*. *AD* = 800 mm, *AB* = 300 mm, *BC* = 700 mm and *CD* = 400 mm.

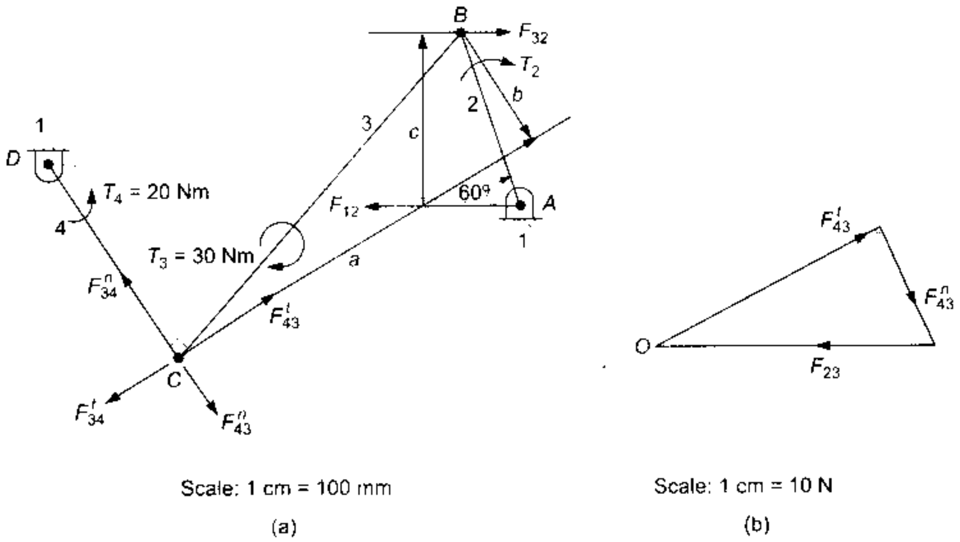


Fig.13.12 Four-bar chain

■ Solution

Taking moments about joint D, we get

$$F_{34}^t \times CD = T_4 = 20$$

$$F_{34}^t = \frac{20}{0.04} = 50 \text{ N}$$

$$F_{43}^t = -F_{34}^t$$

$$a = 670 \text{ mm}, \; b = 200 \text{ mm}$$

Taking moments about joint B, we get

$$F_{43}^t \times b + F_{43}^n \times a = T_3$$

$$50 \times 0.2 + F_{43}^n \times 0.67 = 30$$

$$F_{43}^n = \frac{20}{0.67} = 29.85 \text{ N}$$

$$F_{34}^n = -F_{43}^n$$

Draw force polygon for link BC, as shown in Fig.13.12(b). $F_{23} = 59$ N. $F_{32} = -F_{23}$, $F_{12} = F_{32}$, $c = 280$ mm.

$$T_2 = F_{32} \times c = 59 \times 0.28 = 16.52 \text{ Nm cw}$$

Torque exerted by the crankshaft on the crank = 16.52 Nm (ccw)

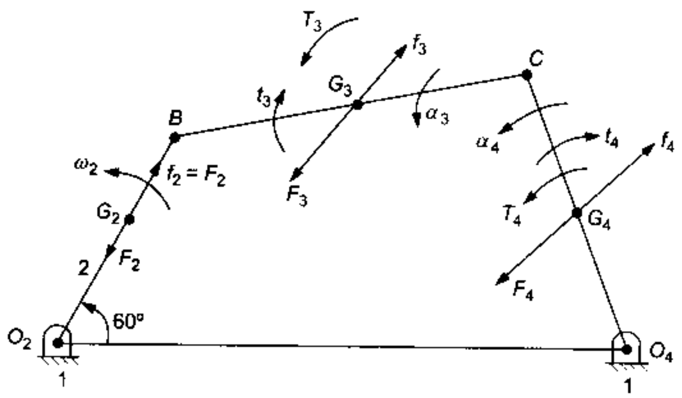
Example 13.4

A four-bar mechanism has the following lengths of various links: $O_2O_4 = 80$ mm, $O_2B = 330$ mm, $BC = 500$ mm, $O_4C = 400$ mm, $O_2G_2 = 200$ mm, $BG_3 = 250$ mm and $O_4G_4 = 200$ mm. The masses of links are $m_2 = 1.2$ kg, $m_3 = 1.5$ kg and $m_4 = 2$ kg. The moment of inertia of links about their C.G. are $I_2 = 0.05$ kgm^2, $I_3 = 0.07$ kgm^2 and $I_4 = 0.02$ kgm^2.

The crank O_2B rotates at 100 rad/s counter-clockwise. Neglecting gravity effects, determine the forces in the joints and the input torque.

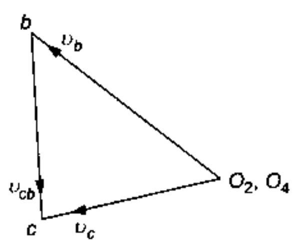
■ Solution

The mechanism has been drawn in Fig.13.13(a). $\omega_2 = 100$ rad/s. $v_b = 100 \times 0.33 = 33$ m/s. Draw the velocity diagram as shown in Fig.13.13(b). $v_c = 25$ m/s, $v_{cb} = 26$ m/s.



Scale: 1 cm = 100 mm

(a)



Scale: 1 cm = 10 m/s

(b) Velocity diagram

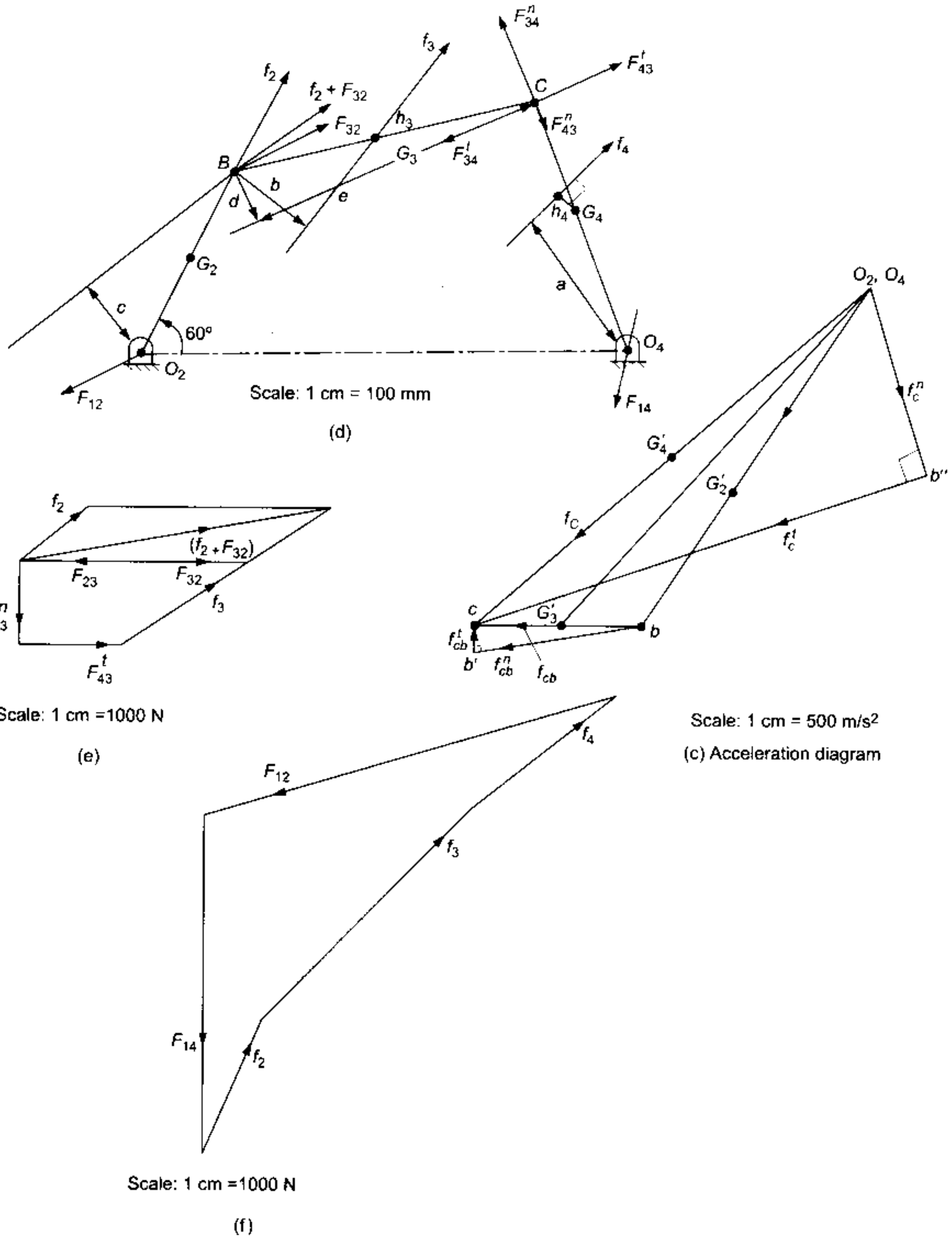


Fig.13.13 Four-bar mechanism

$$a_b^n = \frac{v_b^2}{O_2B} = \frac{33^2}{0.33} = 3300 \text{ m/s}^2$$

$$a_{cb}^n = \frac{v_{cb}^2}{BC} = \frac{26^2}{0.5} = 1352 \text{ m/s}^2$$

$$a_c^n = \frac{v_c^2}{O_4C} = \frac{25^2}{0.4} = 1562.5 \text{ m/s}^2$$

Draw the acceleration diagram, as shown in Fig.13.13(c). $o_2b = 6.6$ cm, $bc = 2.8$ cm, $o_4c = 8.4$ cm.

$$o_2G'_2 = \frac{O_2G_2 \times o_2b}{O_2B} = \frac{200 \times 6.6}{330} = 4 \text{ cm}$$

$$bG'_3 = \frac{BG_3 \times bc}{BC} = \frac{250 \times 2.8}{500} = 1.4 \text{ cm}$$

$$o_4G'_4 = \frac{O_4G_4 \times o_4c}{O_4C} = \frac{200 \times 8.4}{400} = 4.2 \text{ cm}$$

Acceleration of G_2, $a_{G2} = o_2G'_2 = 4 \times 500 = 2000 \text{ m/s}^2$

Acceleration of G_3, $a_{G3} = o_2G'_3 = 7.4 \times 500 = 3700 \text{ m/s}^2$

Acceleration of G_4, $a_{G4} = o_4G'_4 = 4.2 \times 500 = 2100 \text{ m/s}^2$

$$a_{cb}^t = b'c = 0.5 \text{ cm} = 250 \text{ m/s}^2$$

$$a_c^t = b''c = 7.8 \text{ cm} = 3900 \text{ m/s}^2$$

$$\alpha_3 = \frac{a_{cb}^t}{BC} = \frac{250}{0.5} = 500 \text{ rad/s}^2, \text{ (ccw)}$$

$$\alpha_4 = \frac{a_c^t}{O_4C} = \frac{3900}{0.4} = 9750 \text{ rad/s}^2, \text{ (ccw)}$$

$$F_2 = m_2a_{G2} = 1.2 \times 2000 = 2400 \text{ N}$$

Inertia force, $f_2 = -F_2$

$$F_3 = m_3a_{G3} = 1.5 \times 3700 = 4550 \text{ N}$$

Inertia force, $f_3 = -F_3$

$$T_3 = I_3\alpha_3 = 0.07 \times 500 = 35 \text{ Nm (ccw)}$$

Inertia torque, $t_3 = -T_3 = 35$ Nm (cw)

$$F_4 = m_4a_{G4} = 2 \times 2100 = 4200 \text{ N}$$

Inertia force, $f_4 = -F_4$

$$T_4 = I_4\alpha_4 = 0.02 \times 9750 = 195 \text{ Nm (ccw)}$$

Inertia torque, $t_4 = -T_4 = 195$ Nm (cw)

$$h_3 = \frac{t_3}{f_3} = \frac{35}{4550} = 7.7 \text{ mm}$$

$$h_4 = \frac{t_4}{f_4} = \frac{195}{4200} = 46 \text{ mm}$$

The forces and perpendicular distances are shown in Fig.13.13(d).
$a = 230$ mm, $b = 150$ mm, $d = 90$ mm, $e = 490$ mm

$$F'_{34} = \frac{f_4 a}{O_4C} = \frac{4200 \times 230}{400} = 2415 \text{ N}$$
$$F'_{43} = -F'_{34} = -2415 \text{ N}$$

Taking moments about B, we have

$$f_3 b + F'_{43} d - F''_{43} e = 0$$
$$4550 \times 150 + 2415 \times 90 = F''_{43} \times 490$$
$$F''_{43} = 1836.4 \text{ N}$$

Draw the force polygon for link BC, as shown in Fig.13.13(e).

$$F_{23} = 6500 \text{ N}$$
$$F_{32} = -F_{23}$$
$$F_{12} = F_{32}$$

The resultant of f_2 and F_{32} has been obtained in Fig.13.13(e) and is equal to $f_2 + F_{32} = 8600$ N.

$$T_2 = (f_2 + F_{32})c$$
$$= 8600 \times 0.13 = 1118 \text{ Nm (cw)}$$

Torque exerted by the crankshaft on crank $O_2B = -T_2 = 1118$ Nm (ccw).
Now draw the force polygon for the whole mechanism, as shown in Fig.13.13(f).

$$F_{14} = 5600 \text{ N}$$

Exercises

1 In the four-bar linkage shown in Fig.13.14, the shaft at O_2 exerts a torque of 0.6 Nm clockwise on link 2. Also there is a 45 N force acting vertically downward on link 3 midway between B and C. Determine the resisting torque which the shaft at O_4 exerts on crank 4 and find the forces exerted on the frame at O_2 and O_4. $O_2O_4 = 90$ mm, $O_2B = 50$ mm, $BC = 55$ mm, $O_4C = 30$ mm and $BD = DC = 27.5$ mm.

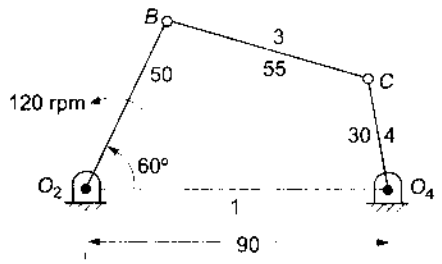


Fig.13.14

2 The slider–crank mechanism of a single cylinder diesel engine is shown in Fig.13.15. A gas force P = 17800 N acts to the left through piston pin C. The crank rotates counter-clockwise at a constant speed of 1800 rpm. Determine (a) the forces F_{14} and F_{12} and the torque T_2 exerted by the crankshaft on the crank for equilibrium and (b) the magnitude and direction of the shaking force and its location from point O_2. O_2B = 75 mm, O_2G_2 = 50 mm, BC = 280 mm and BG_3 = 125 mm; m_2 = 2.25 kg, m_3 = 3.65 kg and m_4 = 2.75 kg; i_2 = 0.0055 kgm^2 and i_3 = 0.041 kgm^2.

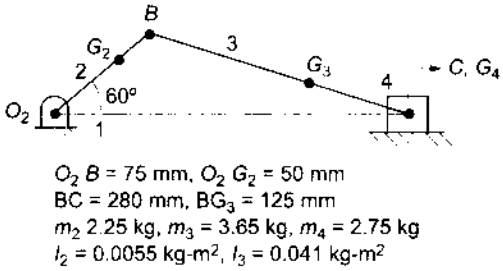


Fig.13.15

3 The crank of a four-bar mechanism shown in Fig.13.16 is balanced and rotating in the counter-clockwise direction at a constant angular speed of 200 rad/s. The particulars of the mechanism are: O_2A = 50 mm, AB = 450 mm, AG_3 = 225 mm; O_4B = 200 mm, O_4G_4 = 100 mm and O_2O_4 = 350 mm; W_3 = 1.2 kg and W_4 = 3 kg; I_3 = 68.6 kgcm^2 and I_4 = 55 kgcm^2. G_3 and G_4 are mass-centres of links 3 and 4, W_3 and W_4 their respective masses and I_3 and I_4 their respective mass moment of inertia about their mass-centres. For the given angular position of the crank 2, draw the velocity and acceleration diagrams and find the angular accelerations of links 3 and 4. Determine also the forces acting at the pin joints A and B and the external torque which must be applied to link 2. Ignore the effect of gravitation.

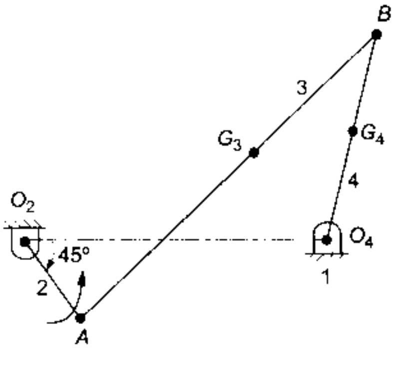


Fig.13.16

4 The lengths of the links of a four-bar chain shown in Fig.13.17 are AB = 60 mm, BC = 180 mm, CD = 110 mm and AD = 200 mm. Link AD is fixed and AB turns at a uniform speed of 180 rpm ccw. The mass of link BC is 2.5 kg, it's centre of gravity is 100 m from C and its radius of gyration about an axis through the centre of gravity is 75 mm. The mass of link CD is 1.5 kg, its centre of gravity is 40 mm from C and its radius of gyration about an axis through D is 80 mm. When BA is at right angles to AD and B and C lie on opposite sides of AD, find the torque on AB to overcome the inertia of the links and the forces which act on the pins at B and C. Neglect gravity effects.

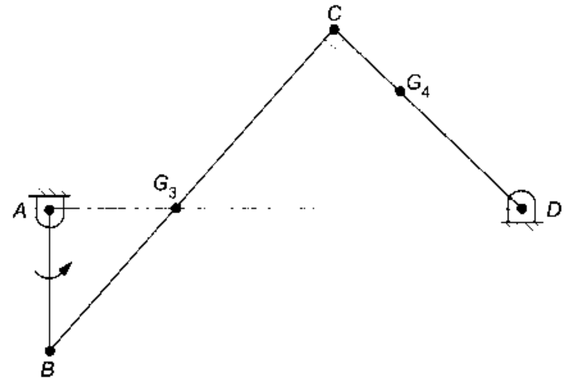


Fig.13.17

5 The crank of a four-bar chain shown in Fig.13.18 is rotating at a speed of 24 rad/s in a clockwise direction. The particulars of the chain are O_2A = 75 mm, AB = 250 mm, O_4B = 250 mm, AG_3 = 125 mm, O_2G_4 = 150 mm and BC = 130 mm; m_2 = 4.5 kg, m_3 = 2 kg and m_4 = 4 kg, I_2 = 0.025 kgm^2, I_3 = 0.008 kgm^2 and I_4 = 0.035 kgm^2. The mass moment of inertias are about the respective mass-centres. Determine the forces acting at the pin joints A, B and the external torque which must be applied to link 2. Ignore the effect of gravitation.

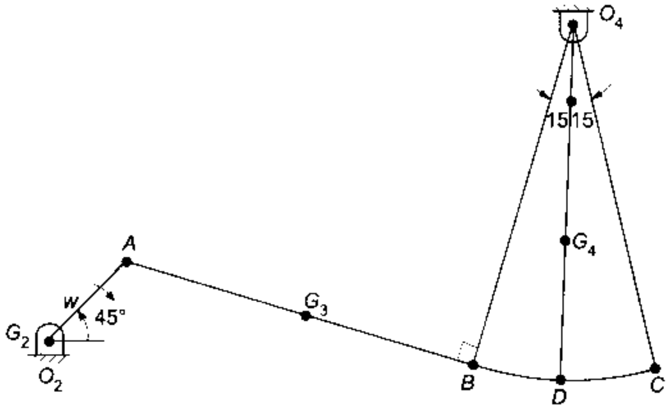
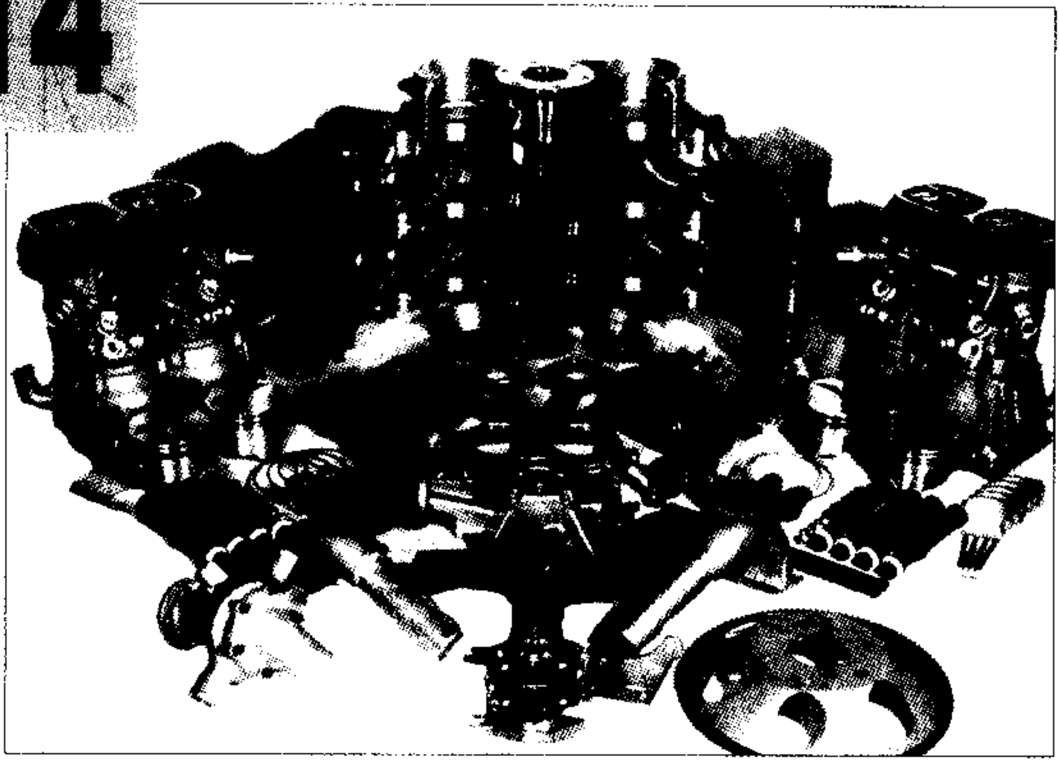


Fig.13.18

14



GYROSCOPIC AND PRECESSIONAL MOTION

14.1 INTRODUCTION

In vehicles having engines with rotating parts of high moment of inertia, gyroscopic forces are in action when the vehicle is changing direction of motion. When Automotive vehicles turn with high velocities, gyroscopic forces act on spinning parts such as crankshaft, flywheel, clutch, transmission gears, propeller shaft and wheels. Engine parts as well as the propeller and the gear reduction system of an airplane are under the influence of gyroscopic effects in turns and pullouts. Locomotives and ships are similarly affected. In this chapter, we shall study the gyroscopic and precessional motion of road vehicles, aeroplanes and ships.

14.2 PRECESSIONAL MOTION

Consider a plane disc spinning about the axis *OX* with angular speed ω, as shown in Fig.14.1(a). After a short interval of time δt, let the disc be spinning with angular speed $\omega + \delta\omega$ about the new axis *OX'* inclined at a small angle $\delta\theta$ with *OX*. The angular speed ω is represented by vector *OX* and $\omega + \delta\omega$ by vector *OX'* in Fig.14.1(b). The vector *XX'* represents the change of angular speed in time δt.

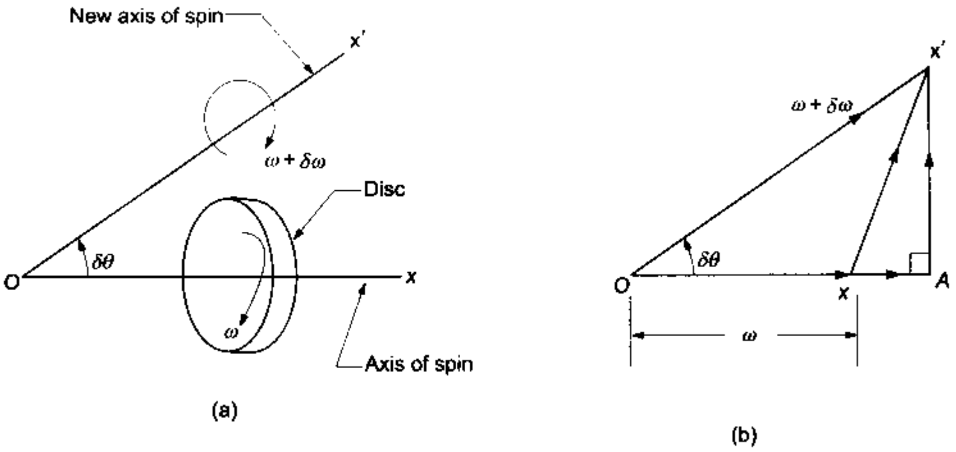


Fig.14.1 Precessional motion

Angular acceleration along

$$OX = \lim_{\delta t \to 0}\left[\frac{(\omega + \delta\omega)\cos\delta\theta - \omega}{\delta t}\right]$$

$$= \lim_{\delta t \to 0}\left[\frac{\omega\cos\delta\theta + \delta\omega\cos\delta\theta - \omega}{\delta t}\right]$$

$$= \lim_{\delta t \to 0}\left[\frac{\omega + \delta\omega - \omega}{\delta t}\right] \qquad \text{(Because} \quad \cos\delta\theta \approx 1)$$

$$= \lim_{\delta t \to 0}\frac{\delta\omega}{\delta t}$$

$$= \frac{d\omega}{dt} \qquad (14.1)$$

Angular (or gyroscopic) acceleration perpendicular to OX

$$= \lim_{\delta t \to 0}\left[\frac{(\omega + \delta\omega)\sin\delta\theta}{\delta t}\right]$$

$$= \lim_{\delta t \to 0}\left[\frac{\omega\cdot\delta\theta + \delta\omega\cdot\delta\theta}{\delta t}\right]$$

$$= \frac{\omega\cdot d\theta}{dt} \quad \text{(Neglecting } \delta\omega\cdot\delta\theta\text{, being small)}$$

$$= \omega\cdot\omega_p \qquad (14.2)$$

where ω_p is the precessional angular speed of the spin axis.

Total angular acceleration of disc $= XX'$

$$= XA + AX'$$

$$= \frac{d\omega}{dt} \nrightarrow \frac{\omega\cdot d\theta}{dt}$$

$$= \frac{d\omega}{dt} \nrightarrow \omega\cdot\omega_p \qquad (14.3)$$

where $\nrightarrow$ means vector sum.

14.3 DEFINITIONS

Gyroscope A gyroscope is a body which while spinning about an axis is free to move in other direction under the action of external forces.

Axis of spin The axis of spin is the axis about which the body revolves.

Gyroscopic effect Consider a body spinning about an axis *OX* (Fig. 14.2). If a couple represented by a vector *OZ* perpendicular to *OX* is applied, then the body tries to precess about an axis *OY* which is perpendicular to both *OX* and *OZ*. This combined effect is called gyroscopic or precessional effect. The plane of spin, plane of precession and plane of gyroscopic couple are mutually perpendicular.

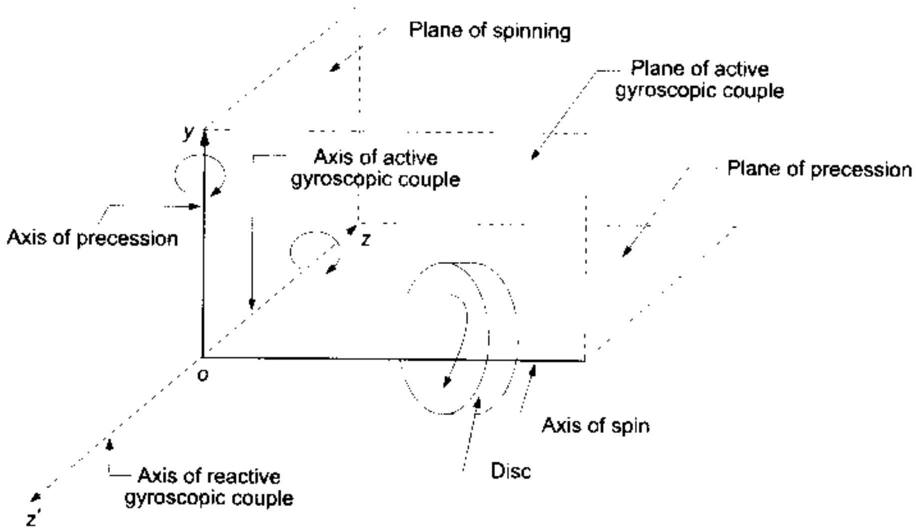


Fig.14.2 Gyroscopic definitions

Precession Precession means the rotation about the third axis *OY* which is perpendicular to both the spin axis *OX* and the couple axis *OZ*.

Axis of precession The axis of precession is the third axis *OY* about which a body revolves and is perpendicular to both the spin axis *OX* and couple axis *OZ*, is called the axis of precession.

14.4 GYROSCOPIC COUPLE OF A PLANE DISC

Consider a plane disc of moment of inertia I spinning with angular speed ω about the spinning axis, as shown in Fig. 14.3(a). The angular momentum H of the spinning disc is,

$$H = I\omega$$

The rate of change of angular momentum with respect to time is proportional to the applied couple C.

$$C = I\alpha = I\frac{d\omega}{dt}$$

$$= \frac{d}{dt}(I\omega) = \frac{dH}{dt} \qquad (14.4)$$

where α is the angular acceleration.

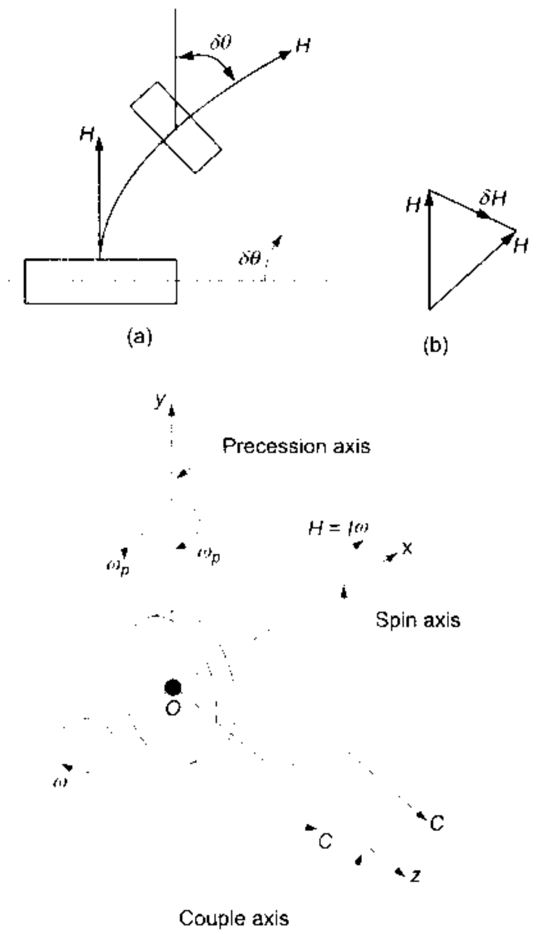


Fig.14.3 Gyroscopic couple

If the spin axis is made to change angular position, gyroscopic action results. For constant ω, the magnitude of the angular momentum remains constant for an angular displacement $\delta\theta$ of the spin axis. However, a change in angular momentum δH exists because of the change in direction of the momentum, as shown in Fig.14.3(b).

$$\delta H = (I\omega)\delta\theta$$

$$\lim_{\delta t \to 0} \frac{\delta H}{\delta t} = \lim_{\delta t \to 0} (I\omega) \cdot \left(\frac{\delta\theta}{\delta t}\right)$$

$$\frac{\delta H}{dt} = I\omega \cdot \frac{d\theta}{dt}$$

$$C = I\omega \cdot \omega_p \tag{14.5}$$

Fig.14.3 shows the X-axis as the spin axis and the Y-axis as the precession axis. The Z-axis becomes the couple axis.

Example 14.1

Determine the gyroscopic couple of a 3 m diameter solid aluminium alloy four-bladed propeller in which each blade has a mass of 20 kg. The test manoeuver of the airplane is a power-on flat spin in which the propeller speed is 1500 rpm and the rotation of the flat spin is 1 rad/s. The radius of gyration of the propeller with respect to the propeller axis is approximately half of the propeller radius.

■ **Solution**

Radius of gyration, $K = r_m = 0.5 \times 3 = 1.5$ m

Moment of inertia, $I = MK^2$

$= 20 \times (1.5)^2 = 45$ kgm^2

Angular speed, $\omega = 2\pi \times \dfrac{1500}{60} = 157.08$ rad/s

Precessional speed, $\omega_p = 1$ rad/s

Couple, $C = I\omega \cdot \omega_p$

$= 45 \times 157.08 \times 1$

$= 7068.6$ Nm

Example 14.2

A boat is propelled by a steam turbine. The moment of inertia of the rotor, shaft and propeller is 60 kgm^2. The turbine runs at 3000 rpm in clockwise direction looking from the front. The boat describes a circular path towards the right making one revolution in 10 seconds. Find the magnitude and direction of the couple acting on the boat hull.

■ **Solution**

$$\omega = \frac{2\pi N}{60} = 2\pi \times \frac{3000}{60} = 314.16 \text{ rad/s}$$

$$\omega_p = \frac{2\pi}{10} = 0.628 \text{ rad/s}$$

Applied couple, $C = I \cdot \omega \cdot \omega_p$

$= 60 \times 314.16 \times 0.628$

$= 11837.5$ Nm

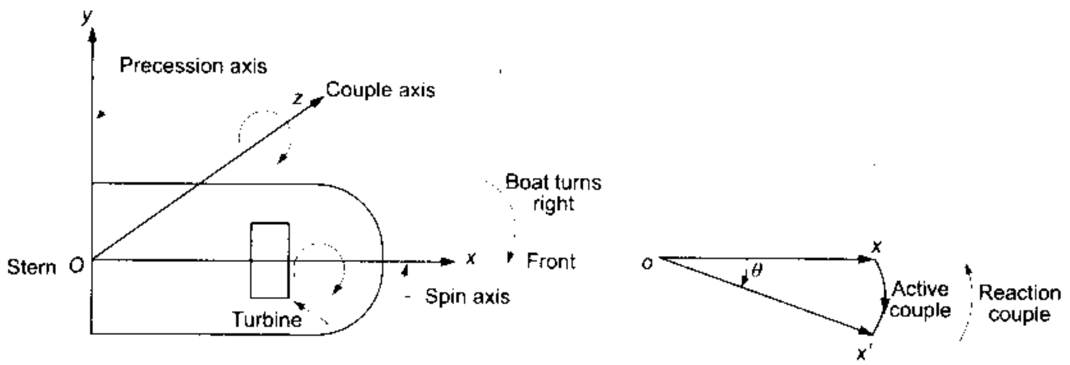


Fig.14.4 Gyroscopic effect on a boat

The vector diagram of the gyroscopic effect is shown in Fig.14.4. The applied couple will lower the front and raise the stern. The reaction couple $X'X$ will raise the front and lower the hind of the boat.

Example 14.3

A uniform disc of 100 mm diameter and 5 kg mass is mounted midway between bearings 100 mm apart, which keeps it in a horizontal plane. The disc spins about its axis with a constant speed of 1200 rpm, as shown in Fig.14.5(a). Find the resultant reaction at each bearing due to the mass and gyroscopic effects.

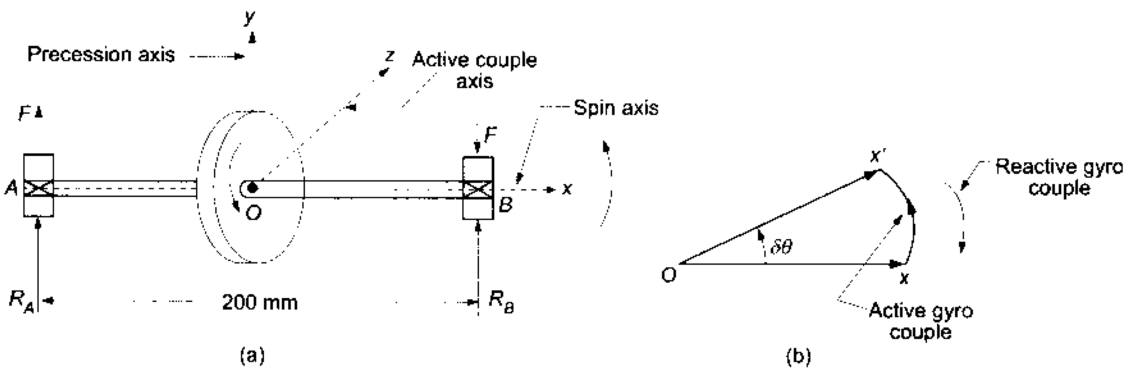


Fig.14.5 Gyroscopic couple

■ **Solution**

$$\omega = 2\pi \times \frac{1200}{60} = 125.66 \text{ rad/s}$$

$$\omega_p = 2\pi \times \frac{50}{60} = 5.236 \text{ rad/s}$$

$$I = 0.5mr^2 = 0.5 \times 5\left(50 \times 10^{-3}\right)^2 = 0.00625 \text{ kgm}^2$$

$$\begin{aligned} C &= I \cdot \omega \cdot \omega_p \\ &= 0.00625 \times 125.66 \times 5.236 \\ &= 4.112 \text{ Nm} \end{aligned}$$

The direction of reaction gyroscopic couple is shown in Fig.14.5(b).

Bearing reactions

(a) Due to self weight of the disc

$$R_A = R_B = \frac{5 \times 9.81}{2} = 24.525 \text{ N}$$

(b) Due to reaction gyroscopic couple

$$F = \frac{C}{0.1}$$

$$R_A = F = \frac{4.112}{0.1} = 41.12 \text{ N} \uparrow$$

$$R_B = -F = -41.12 \text{ N} \downarrow$$

Resultant bearing reactions

$$R_A = 24.525 + 41.12 = 65.645 \text{ N} \uparrow$$
$$R_B = 24.525 - 41.12 = -16.515 \text{ N} \downarrow$$

14.5 GYROSCOPIC COUPLE ON AN AREOPLANE

The top and front views of an aeroplane taking a left turn are shown in Fig.14.6(a) and (b) respectively. Let the propeller rotate clockwise as seen from the rear (or tail end).

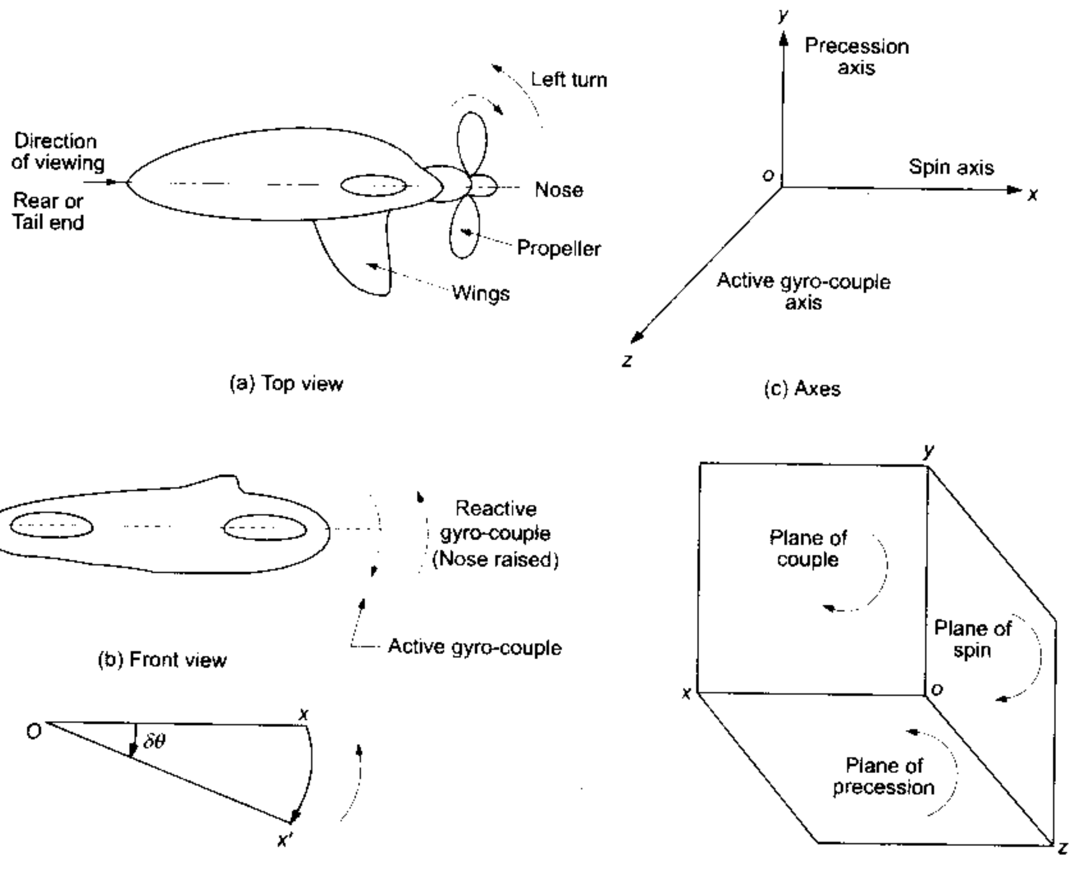


Fig.14.6 Gyroscopic effect of aeroplane

Gyroscopic couple acting on the aeroplane, $C = I \cdot \omega \cdot \omega_p$

where I = moment of inertia of the engine and propeller

$= mK^2$

m = mass of the engine and propeller

K = radius of gyration

ω = angular speed of the engine

ω_p = angular speed of precession = v/R
v = linear velocity of the aeroplane
R = radius of curvature.

As discussed earlier, the reactive gyro-couple will be counter-clockwise, which will raise the nose and dip the tail of the aeroplane.

Example 14.4

An aeroplane makes a complete half circle radius towards left when flying at 210 km/h. The rotary engine and the propeller of the plane is of 50 kg mass having a radius of gyration of 300 mm. The engine rotates at 2400 rpm clockwise as seen from the rear. Find the gyroscopic couple on the aircraft and its effect on the plane.

■ **Solution**

$$\omega = 2\pi \times \frac{2400}{60} = 251.33 \text{ rad/s}$$

$$\omega_p = \frac{v}{R} = \frac{210 \times 10^3}{3600 \times 60} = 0.972 \text{ rad/s}$$

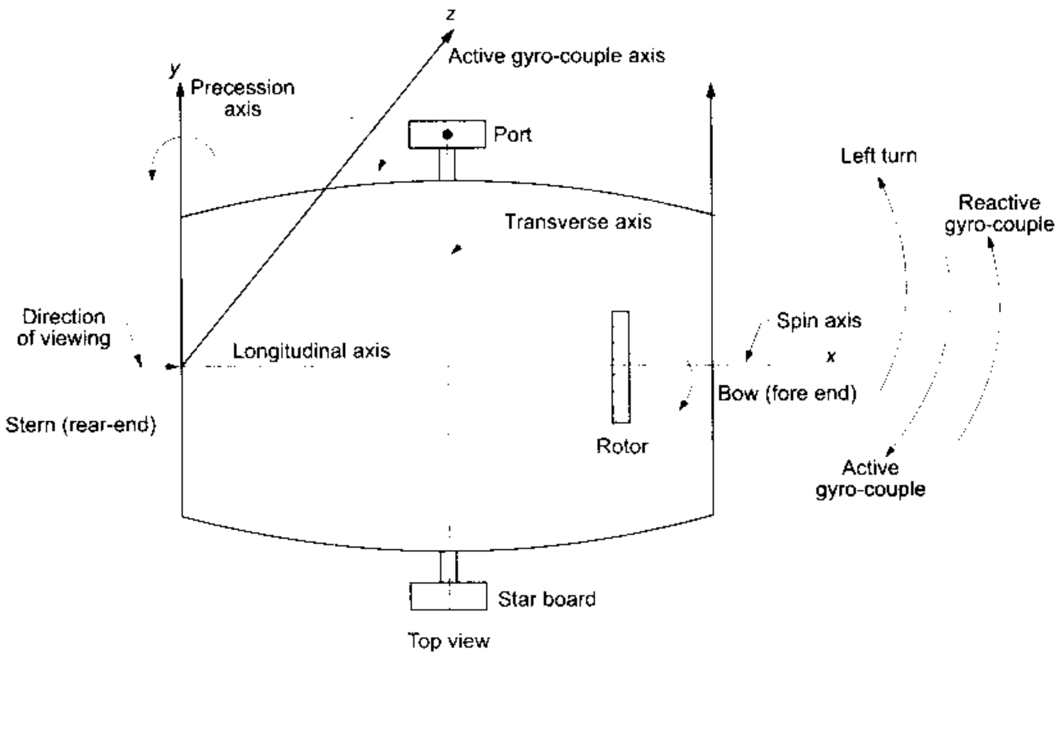
$$I = mK^2 = 50 \times (0.3)^2 = 4.5 \text{ kgm}^2$$

$$C = I \cdot \omega \cdot \omega_p = 4.5 \times 251.33 \times 0.972 = 1099.32 \text{ Nm}$$

The reaction gyro-couple will raise the nose and dip the tail.

14.6 GYROSCOPIC EFFECTS ON A NAVAL SHIP

The following terms for a naval ship in reference to Fig.14.7 are defined:



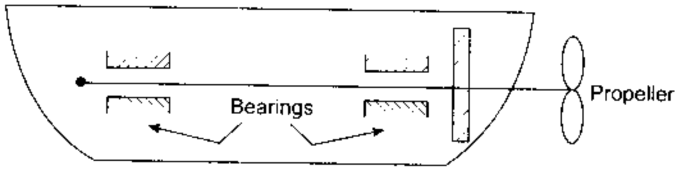


Fig.14.7 Gyroscopic effect on a naval ship

Bow Bow is the fore-end of the ship.

Stern Stern is the rear-end of the ship.

Starboard Starboard is the right hand side of the ship while looking in the direction of motion.

Port Port is the left hand side of the ship while looking in the direction of motion.

Steering Steering is the turning of the ship in a curve while moving forward.

Pitching Pitching is the moving of the ship up and down the horizontal position in a vertical plane about transverse axis.

Rolling Rolling is the sideway motion of the ship about longitudinal axis.

Steering The gyroscopic effect on a naval ship during steering can be obtained as explained for the aeroplane Table 14.1 may be used to determine the gyroscopic effects.

Table 14.1

Direction of steering	Direction of rotor rotation (viewed from stern)	Bow	Stern
Left	CW	Raised	Lowered
Right	CW	Lowered	Raised
Left	CCW	Lowered	Raised
Right	CCW	Raised	Lowered

Pitching Pitching of the naval ship is assumed to take place with simple harmonic motion. The pitching angle at time t is (see Fig.14.8),

$$\theta = A \sin \omega_o t$$

where A = amplitude of swing in radians

ω_o = angular velocity of simple harmonic motion

$= \dfrac{2\pi}{t_p}$

t_p = time period of pitching.

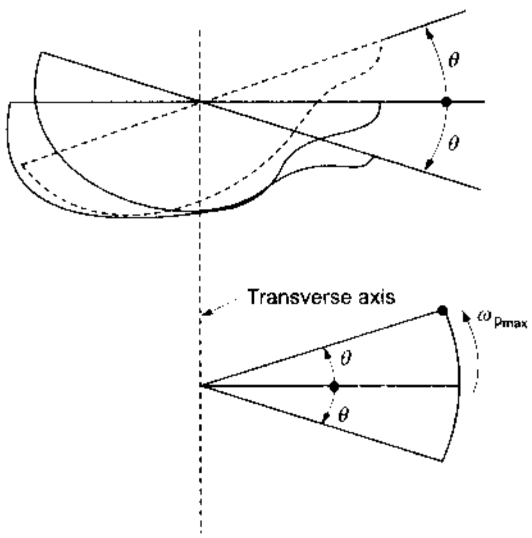


Fig.14.8 Pitching of naval ship

Angular velocity of precession,

$$\omega_p = \frac{d\theta}{dt} = A\omega_o \cos \omega_o t$$

$$(\omega_p)_{\max} = A\omega_o = A \times \frac{2\pi}{t_p} \quad \text{for} \quad \cos \omega_o t = 1$$

Maximum gyro-couple, $$C_{\max} = I \cdot \omega \cdot (\omega_p)_{\max} \tag{14.6}$$

where I = moment of inertia of turbine rotor and other rotating masses of the naval ship.
ω = angular velocity of rotating masses.

The effects of pitching are as follows:

1. When the pitching is upward, the gyroscopic effect will try to move the ship towards starboard.
2. On the other hand, if the pitching is downward, the gyroscopic effect is to turn the ship towards port side.
3. The pitching of a ship produces forces on the bearings which act horizontally and perpendicular to the motion of the ship.
4. The maximum gyroscopic couple tends to shear the holding down bolts.

Angular acceleration during pitching,

$$\alpha = \frac{d^2\theta}{dt^2} = -A\omega_o^2 \sin \omega_o t$$

$$\alpha_{\max} = A\omega_o^2 = A\left(\frac{2\pi}{t_p}\right)^2 \tag{14.7}$$

Rolling The axis of rolling and that of the rotor of the turbine are generally the same. So, there is no precession of axis of spin and there is no gyroscopic effect during rolling of the naval ship.

14.6.1 Ship Stabilization

A naval ship is normally stable, but it requires stabilization when it has to face heavy sea. The ship will either pitch or roll. The amplitude of rolling is much higher than that of pitching. The ship in such a case is stabilized by producing couples in the opposite direction to that of the disturbing couples which are applied by the waves on the ship.

Example 14.5

A ship is propelled by a turbine rotor of mass 500 kg and has a speed of 2400 rpm. The rotor has a radius of gyration of 0.5 m and rotates in clockwise direction when viewed from stern. Find the gyroscopic effects in the following cases:

(a) The ship runs at a speed of 15 knots (1 knot = 1860 m/h). It steers to the left in a curve of 60 m radius.

(b) The spin pitches ±5° from the horizontal position with the time period of 20 s of simple harmonic motion.

(c) The ship rolls with angular velocity of 0.04 rad/s clockwise when viewed from stern.

Also calculate the maximum acceleration during pitching.

■ Solution

(a)

$$\omega = 2\pi \times \frac{2400}{60} = 251.3 \quad \text{rad/s}$$

$$I = mK^2 = 500 \times (0.5)^2 = 125 \text{ kgm}^2$$

$$\omega_p = \frac{15 \times 1860}{3600 \times 60} = 0.129 \text{ rad/s}$$

$$C = I \cdot \omega \cdot \omega_p$$

$$= 125 \times 251.3 \times 0.129 = 4052.2 \text{ N-m}$$

(b)

$$\omega_o = \frac{2\pi}{t_p} = \frac{2\pi}{20} = 0.314 \text{ rad/s}$$

$$(\omega_p)_{\max} = A\omega_o = \frac{5\pi}{180} \times 0.314 = 0.0274 \text{ rad/s}$$

$$C_{\max} = I \cdot \omega \cdot (\omega_p)_{\max}$$

$$= 125 \times 251.3 \times 0.0274 = 860.7 \text{ N-m}$$

(c)

$$\omega_p = 0.04 \text{ rad/s}$$

$$C = I \cdot \omega \cdot \omega_p = 125 \times 251.3 \times 0.04 = 1256.5 \text{ N-m}$$

$$\alpha_{\max} = A\omega_o^2 = \frac{5\pi}{180} \times (0.314)^2 = 0.0086 \text{ rad/s}^2$$

14.7 STABILITY OF A FOUR-WHEEL VEHICLE TAKING A TURN

Consider a four-wheel vehicle of weight W taking a left turn as shown in Fig.14.9. The weight W is assumed to be distributed equally on all the four wheels. Therefore, the load on each of the four wheels A, B, C, and D is $W/4$. Let R be the radius of curvature of the curved path, assumed to be larger than the radius r of the wheels. Let the centre of gravity G of the vehicle be at a height h from the level of the road and the width of track $AB = CD = a$. The forces accounting for the stability of the vehicle are as follows:

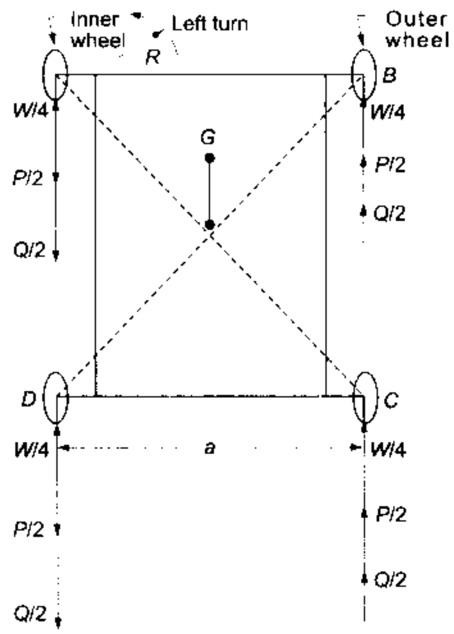


Fig.14.9 Four wheel vehicle taking a left turn

1. Weight of the vehicle W, giving rise to upward reaction of $W/4$ at each wheel
2. Precession of vehicle

Let I_w = moment of inertia of each wheel

I_e = moment of inertia of engine, flywheel, etc.

ω_w = velocity of spin of each wheel about its own axis

ω_e = velocity of spin of engine, flywheel etc.

ω_p = angular velocity of precession = v/R

v = linear velocity of vehicle

Gyroscopic couple due to four wheels,

$$C_w = 4I_w \cdot \omega_w \cdot \omega_p$$

Gyroscopic couple due to other rotating parts of the engine, like flywheel etc.,

$$C_e = I_e \cdot \omega_e \cdot \omega_p$$

Total gyroscopic couple,

$$\begin{aligned} C &= C_w \pm C_e \\ &= 4I_w \cdot \omega_w \cdot \omega_p \pm I_e \cdot \omega_e \cdot \omega_p \\ &= 4I_w \cdot \omega_w \cdot \omega_p \pm I_e \cdot i\omega_w \cdot \omega_p \end{aligned} \quad (14.8)$$

where $i = \dfrac{\omega_e}{\omega_w}$ is the gear ratio of engine rotating parts to wheel.

Use the +ve sign when the wheel and engine rotating parts rotate in the same direction, otherwise use the −ve sign. A vertical reaction will be produced due to this gyroscopic couple. The reaction will be positive at the outer wheels and negative on the inner wheels.

Magnitude of vertical reaction at each of the outer or inner wheels,

$$\frac{P}{2} = \frac{C}{2a} \tag{14.9}$$

3. Centrifugal effect

Centrifugal force acting outward at the centre of the vehicle,

$$F_c = \frac{Wv^2}{gR}$$

Couple tending to overturn the wheels,

$$C_c = F_c.h = \frac{Wv^2h}{gR}$$

This couple is balanced by the vertical reactions at the four wheels, being positive at the outer and negative at the inner wheels.

Vertical reaction at each of outer or inner wheel,

$$\frac{Q}{2} = \frac{C_c}{2a} \tag{14.10}$$

Total vertical reaction at each inner wheel,

$$P_i = \frac{W}{2} - \frac{P}{2} - \frac{Q}{2} \tag{14.11}$$

Total vertical reaction at each outer wheel,

$$P_o = \frac{W}{2} + \frac{P}{2} + \frac{Q}{2} \tag{14.12}$$

When P_i is zero or negative, the tyres of the vehicle inner wheels will leave the ground tending to overturn the vehicle.

For $\quad P_i \leq 0, \; \frac{W}{2} \leq \frac{P+Q}{2}$

$$\text{or} \quad W \leq (P + Q) \tag{14.13}$$

Thus, the vehicle may overturn, when

1. ω_w is high, that is, the vehicle is running at a high speed.
2. h is high, that is, the loaded vehicle is sufficiently high above the ground.
3. R is small, that is, the vehicle is taking a sharp turn.
4. W is large, that is, the vehicle is overloaded.

In order to reduce the total gyroscopic couple, the engine must be provided with a heavy flywheel which should rotate in the opposite direction to that of the wheels.

Example 14.6

A motor car negotiates a curve of 40 m radius at a speed of 60 km/h. Determine the magnitudes of the centrifugal and gyroscopic couples acting on the motor car and state the effect of each of these on the road reactions on the wheels. Assume the following:

(a) Each road wheel has a moment of inertia of 4 kgm^2 and an effective road radius of 0.5 m.

(b) The rotating parts of the engine and transmission are equivalent to a flywheel of mass 80 kg with a radius of gyration of 0.1 m. The engine turns in a clockwise direction when viewed from the front.

(c) The back axle ratio is 4:1 and the drive through the gear box is direct.

(d) The car weighs 10 kN and has its centre of gravity at 0.6 m above the road level. The car takes a right hand turn.

Solution

$$v = 60 \times \frac{1000}{3600} = 16.67 \quad \text{m/s}$$

Angular velocity of wheel,

$$\omega_w = \frac{v}{r} = \frac{16.67}{0.5} = 314.33 \text{ rad/s}$$

Angular velocity of precession of wheels,

$$\omega_{pw} = \frac{v}{R} = \frac{16.67}{40} = 0.417 \text{ rad/s}$$

(a) Gyroscopic couple due to wheels,

$$C_w = 4I_w \cdot \omega_w \cdot \omega_{pw}$$
$$= 4 \times 4 \times 314.33 \times 0.417 = 223 \text{ Nm}$$

The reaction gyro-couple due to wheels will tend to lift the inner wheels and depress the outer wheels.

(b) Gyroscopic couple due to engine rotating parts,

$$C_e = I_e \cdot \omega_e \cdot \omega_{pe}$$
$$= m_e K^2 \times i\omega_w \cdot \omega_{pe}$$
$$= 80 \times 0.1^2 \times 4 \times 314.33 \times 0.417$$
$$= 44.48 \text{ Nm}$$

The reaction gyro-couple due to engine rotating parts will tend to lift the front wheels and depress the rear wheels.

(c) Centrifugal force,

$$F_c = \frac{Wv^2}{gR}$$
$$= 10 \times 1000 \times \frac{(16.67)^2}{9.81 \times 40}$$
$$= 7078.95 \text{ N}$$

Centrifugal couple, $C_c = 7078.95 \times 0.6 = 4247.4$ Nm

The reaction centrifugal couple will tend to lift the inner wheels and depress the outer wheels.

Example 14.7

A rear engine automobile is travelling along a track of 100 m mean radius. Each of the four wheels has a moment of inertia of 2 kgm^2 and an effective diameter of 0.6 m. The rotating parts of the engine have a moment of inertia of 1.25 kgm^2. The engine axis is parallel to the rear axle and the crankshaft rotates in the same direction as the wheels. The gear ratio of engine to back axle is 3:1. The automobile mass is 1500 kg and its centre of gravity is 0.5 m above the road level. The width of track of the vehicle is 1.5 m.

Determine the limiting speed of the vehicle around the curve for all four wheels to maintain contact with the road surface if it is not banked.

Solution

Let v = linear speed of the vehicle, m/s

Angular speed of the vehicle,

$$\omega_w = \frac{v}{0.3} \text{ rad/s}$$

Angular speed of engine,

$$\omega_e = \frac{3v}{0.3} = 10v \text{ rad/s}$$

Angular velocity of precession,

$$\omega_{pw} = \frac{v}{R} = \frac{v}{100} \text{ rad/s}$$

Gyroscopic couple,

$$\begin{aligned} C_g &= 4I_w \cdot \omega_w \cdot \omega_{pw} + I_e \cdot \omega_e \cdot \omega_{pw} \\ &= 4I_w \cdot \frac{v}{0.3} \cdot \frac{v}{100} + I_e \cdot 10v \cdot \frac{v}{100} \\ &= 4 \times 2 \times \frac{v^2}{30} + 1.25 \times \frac{v^2}{10} \\ &= (0.2667 + 0.125)v^2 \\ &= 0.3917v^2 \text{ Nm} \end{aligned}$$

Centrifugal force, $$F_c = \frac{mv^2}{R} = \frac{1500v^2}{100} = 15v^2 \text{ N}$$

Centrifugal couple, $$C_c = F_c \cdot h$$

Maximum lift due to centrifugal couple on one wheel,

$$\begin{aligned} &= F_c \cdot \frac{h}{2a} \\ &= 15v^2 \times \frac{0.5}{3} = 2.5v^2 \text{ N} \end{aligned}$$

Maximum lift due to gyro-couple,

$$\frac{C_g}{2a} = 0.3917\frac{v^2}{3} = 0.1306v^2 \text{ N}$$

For safe driving, weight on one wheel should be greater than the maximum lift.

$$1500 \times \frac{9.81}{4} > (2.5 + 0.1306)v^2$$

or $$v^2 < \frac{1500 \times 9.81}{4 \times 2.6306}$$

$$< 1398.44$$

or $$v < 37.396 \text{ m/s}$$

or $$v < 134.62 \text{ km/h}$$

14.8 STABILITY OF A TWO-WHEEL VEHICLE TAKING A TURN

Consider a two-wheel vehicle taking a right turn as shown in Fig.14.10.

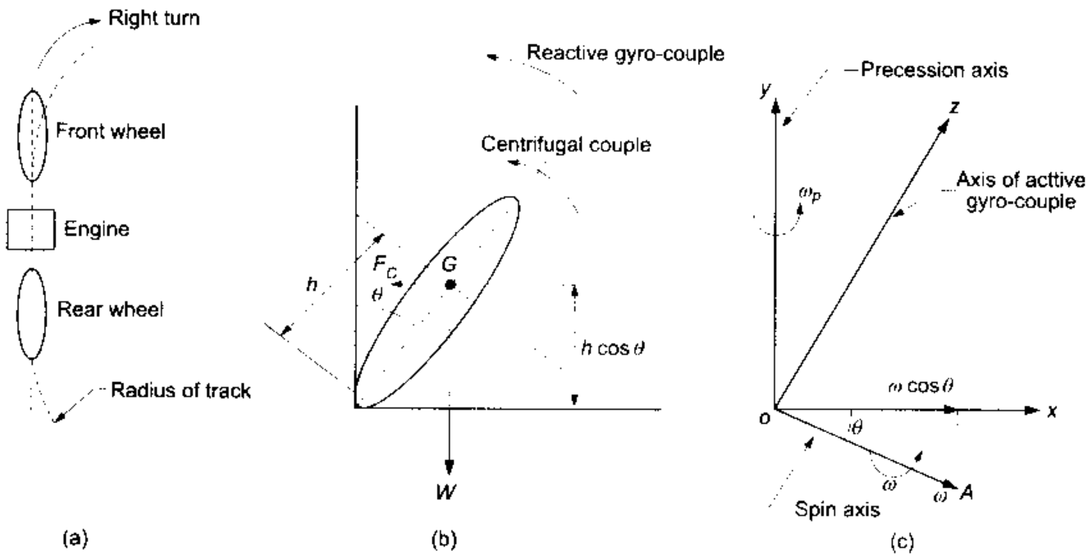


Fig.14.10 Two wheel vehicle taking a turn

Let W = weight of the vehicle and its rider

h = height of the centre of gravity of the vehicle and the rider

r_w = wheel radius

R = track radius

I_w = moment of inertia of each wheel

I_e = moment of inertia of the engine rotating parts

ω_w = angular velocity of the wheels

ω_e = angular velocity of engine the rotating parts

$i = \frac{\omega_e}{\omega_w}$ = gear ratio

v = linear velocity of the vehicle $= r_w \omega_w$

θ = angle of heel or inclination of the vehicle to the vertical

The effects of various forces are as follows:

1. **Gyroscopic couple**

$$v = r_w \omega_w$$

or
$$\omega_w = \frac{v}{r_w}$$

$$\omega_e = i\omega_w = \frac{iv}{r_w}$$

$$\omega_p = \frac{v}{R}$$

Gyroscopic couple,
$$C_g = (2I_w\omega_w \pm I_e\omega_e)\omega_p \cos\theta$$
$$= v^2(2I_w \pm i I_e)\frac{\cos\theta}{r_w R}$$

2. **Centrifugal couple**

Centrifugal force,
$$F_c = \frac{Wv^2}{Rg}$$

Centrifugal couple,
$$C_c = F_c \cdot h\cos\theta$$

Total overturning couple,
$$C_o = C_g + C_c$$
$$= v^2\left[\frac{2I_w \pm i I_e}{r_w} + \frac{Wh}{g}\right]\frac{\cos\theta}{R} \quad (14.14)$$

Balancing couple,
$$C_b = Wh\sin\theta \quad (14.15)$$

For equilibrium of the vehicle (that is no skidding),

$$C_o = C_b$$

$$v^2\left[\frac{2I_w \pm i I_e}{rw} + \frac{Wh}{g}\right]\frac{\cos\theta}{R} = Wh\sin\theta$$

or
$$\tan\theta = \frac{v^2\left[\frac{2I_w \pm i I_e}{rw} + \frac{Wh}{g}\right]}{WhR} \quad (14.16)$$

Example 14.8

The road wheels of a motor cycle have 0.6 m diameter and moment of inertia of 1.5 kgm^2. Its rotating parts have a moment of inertia of 0.3 kgm^2. The speed of engine is six times the speed of wheels and in the same direction, The weight of the motor cycle and its rider is 2 kN and its centre of gravity is 0.6 m above the road level.

Find the heel angle if the motor cycle is travelling at 45 km/h and taking a turn of 30 m radius, when the motor cycle is standing upright and the rider is sitting on it.

Solution

Here $I_w = 1.5$ kgm^2, $I_e = 0.3$ kgm^2, $W = 2$ kN, $h = 0.6$ m, $r_w = 0.3$ m,
$v = 45$ km/h, $i = 6$, $R = 30$ m

$$\tan\theta = \frac{v^2\left[\frac{2I_w+iI_e}{r_w} + \frac{Wh}{g}\right]}{WhR}$$

$$= \left(\frac{45 \times 1000}{3600}\right)^2 \frac{\left[\frac{2\times1.5+6\times0.3}{0.3} + \frac{2000\times0.6}{9.81}\right]}{2000 \times 0.6 \times 30} = 0.60036$$

$$\theta = 30.98^\circ$$

14.9 EFFECT OF PRECESSION ON A DISC FIXED RIGIDLY AT A CERTAIN ANGLE TO A ROTATING SHAFT

Consider a disc fixed rigidly to a rotating shaft (Fig.14.11) at a certain angle such that the polar axis of the disc makes an angle θ with the shaft axis. The shaft revolves with angular speed ω about its axis *OX*. Let *OA* be the diametral axis and *OP* the polar axis of the disc.

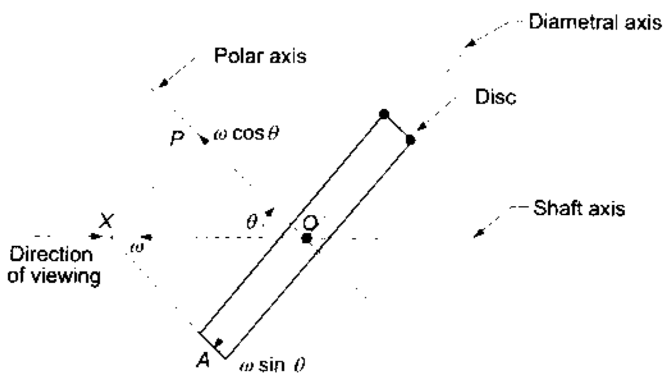


Fig.14.11 Disk fixed rigidly at a certain angle to a rotating shaft

Angular velocity of spin of the disc about $OP = \omega\cos\theta$

Angular velocity of precession about $OA = \omega\sin\theta$

Let I_p = polar moment of inertia of the disc about axis *OP*

Couple producing the precession,

$$C_p = I_p \cdot \omega\cos\theta \cdot \omega\sin\theta$$
$$= 0.5 I_p \omega^2 \sin 2\theta$$

The reaction couple C_p tends to turn the disc in counter-clockwise direction, when viewed from the top, about an axis through *O* in the plane of paper.

Now consider the movement of point *A* about the polar axis *OP*. In this case, *OA* is the axis of spin and *OP* the axis of precession.

Angular velocity of spin about $OA = \omega\sin\theta$

Let I_A = polar moment of inertia of the disc about *OA*

Gyroscopic couple about *OA*,

$$C_A = I_A \cdot \omega\sin\theta \cdot \omega\cos\theta$$
$$= 0.5 I_A \omega^2 \sin 2\theta$$

The effect of this couple will be opposite to that of C_p.
Resultant gyroscopic couple acting on the disc,

$$\begin{aligned} C &= C_p - C_A \\ &= 0.5\omega^2 \sin 2\theta (I_p - I_A) \end{aligned} \tag{14.17}$$

This resultant couple will be acting in counter-clockwise direction, as seen from the top.

Now
$$I_P = \frac{Wr^2}{2g}, \quad \text{where } r = \text{radius of the disc}$$

$$\begin{aligned} I_A &= \frac{W\left(\frac{l^2}{12} + \frac{r^2}{4}\right)}{g}, \quad \text{where } l = \text{width of the disc} \\ &\cong \frac{Wr^2}{4g}, \quad \text{neglecting } l \text{ for a thin disc} \end{aligned}$$

The couple exerted by a thin disc on the shaft,

$$\begin{aligned} C_{\text{disc}} &= W\omega^2 \frac{\sin 2\theta \left(\frac{r^2}{2} - \frac{r^2}{4}\right)}{2g} \\ &= W\omega^2 r^2 \frac{\sin 2\theta}{8g} \end{aligned} \tag{14.18}$$

The shaft tends to turn in the plane of paper in counter-clockwise direction as seen from the top . As a result, the horizontal force is exerted on the bearings.

Example 14.9

A thin disc is fixed to a shaft in such a way that it makes an angle of 2° with a plane at right angles to the axis of the shaft. The disc weighs 25 N and it has a diameter of 0.5 m. If the shaft rotates at 1000 rpm, find the gyroscopic couple acting on the bearing.

■ **Solution**

Here
$$W = 25\text{ N}, r = 0.25\text{ m}, \theta = 2°, \omega = \frac{2\pi \times 1000}{60} = 104.72\text{ rad/s}$$

$$\begin{aligned} C_{\text{disc}} &= W\omega^2 r^2 \frac{\sin 2\theta}{8g} \\ &= 25 \times (104.72)^2 \times (0.25)^2 \times \frac{\sin 4°}{8 \times 9.81} \\ &= 184.45\text{ Nm} \end{aligned}$$

Example 14.10

A wheel of a vehicle travelling on a level track at 80 km/h, falls in a spot hole 15 mm deep and rises again in a total time of 0.15 s. The displacement of the wheel of the vehicle takes place with simple harmonic motion. The diameter of wheel is 1.5 m and the distance between the wheel centers is 1.8 m. The wheel pair with axle have a moment of inertia of 500 kgm^2. Determine the magnitude and gyroscopic effects produced with this phenomenon.

■ **Solution**

$$v = \frac{80 \times 1000}{3600} = 22.22 \text{ m/s}$$

$$I_w = 500 \text{ kgm}^2, r_w = 0.75 \text{ m}, a = 1.8 \text{ m}, h = 15 \text{ mm}, t = 0.15 \text{ s}$$

Amplitude,
$$A_o = \frac{h}{2} = 7.5 \text{ mm}$$

Maximum velocity while falling,

$$v_{max} = \frac{2\pi A_o}{t} = \frac{2\pi \times 7.5 \times 10^{-3}}{0.15} = 0.314 \text{ m/s}$$

$$\omega_p = \frac{v_{max}}{a} = \frac{0.314}{1.8} = 0.174 \text{ rad/s}$$

$$\omega = \frac{v}{r_w} = \frac{22.22}{0.75} = 29.63 \text{ rad/s}$$

$$C = I \cdot \omega \cdot \omega_p$$

$$= 500 \times 29.63 \times 0.174 = 2577.8 \text{ Nm}$$

As the axle goes down, the effect of this is to tend to turn the vehicle towards left as it moves forward .

Example 14.11

A four-wheel vehicle of mass 2500 kg has a wheel base 2.5 m, track width 1.5 m, and height of centre of gravity 0.6 m above the ground level and lies at 1 m from the front axle. Each wheel has an effective diameter of 0.8 m and a moment of inertia of 0.8 kgm^2 .The drive shaft, engine flywheel and transmission are rotating at four times the speed of road wheels, in clockwise direction when viewed from the front, and is equivalent to a mass of 80 kg having a radius of gyration of 100 mm. If the vehicle is taking a right turn of 60 m radius at 60 km/h, find the load on each wheel.

■ **Solution**

Here $m = 2500$ kg, $b = 2.5$ m, $a = 1.5$ m, $h = 0.6$ m, $L = 1$ m, $r_w = 0.4$ m, $I_w = 0.8$ kgm^2, $i = 4$, $m_e = 80$ kg, $K_e = 100$ mm, $R = 60$ m, $v = 60$ km/h.

Let W_1, W_2 = weight on the front and rear wheels respectively.

Taking moments about the front wheels (Fig.14.12), we have

$$2.5 W_2 = 2500 \times 9.81 \times 1$$

$$W_2 = 9810 \text{ N}$$

$$W_1 = 2500 \times 9.81 - 9810 = 14715 \text{ N}$$

Weight on each of the front wheels $= \frac{W_1}{2} = 7357.5$ N

Weight on each of the rear wheels $= \frac{W_2}{2} = 4905$ N

Gyroscopic Effect

$$v = \frac{60 \times 1000}{3600} = 16.67 \text{ m/s}$$

$$I_e = m_e K_e^2 = 80 \times (0.1)^2 = 0.8 \text{ kgm}^2$$

$$\omega_w = \frac{v}{r_w} = \frac{16.67}{0.4} = 41.675 \text{ rad/s}$$

$$\omega_p = \frac{v}{R} = \frac{16.67}{60} = 0.2778 \text{ rad/s}$$

$$C_w = 4I_w \cdot \omega_w \cdot \omega_p$$

$$= 4 \times 0.8 \times 41.675 \times 0.2778 = 37.05 \text{ Nm}$$

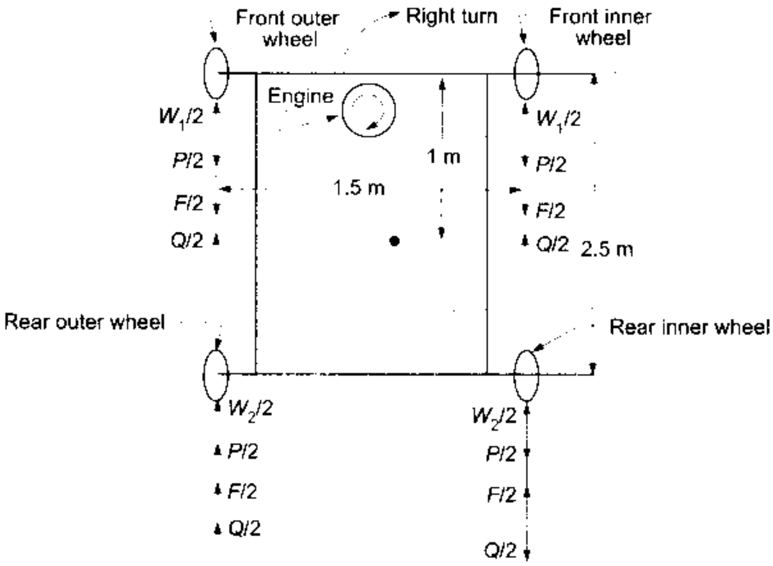


Fig.14.12 Force diagram for wheels

This gyroscopic couple tends to lift the inner wheels and depress the outer wheels. In other words, the reaction will be vertically downward on the inner wheels and vertically upward on the outer wheels. The magnitude of this reaction at each of the inner or outer wheels,

$$\frac{P}{2} = \frac{C_w}{2a} = \frac{37.05}{3} = 12.35 \text{ N}$$

$$C_e = I_e \cdot i\omega_w \cdot \omega_p$$

$$= 0.8 \times 4 \times 41.675 \times 0.2778 = 37.05 \text{ Nm}$$

This gyroscopic couple tends to lift the front wheels and depress the rear wheels. In other words, the reaction will be vertically downwards on the front wheels and vertically upwards on the rear wheels. The magnitude of this reaction at each of the front or rear wheels,

$$\frac{F}{2} = \frac{C_e}{2b} = \frac{37.05}{5} = 7.01 \text{ N}$$

Centrifugal Couple

Centrifugal force,

$$F_c = \frac{mv^2}{R} = \frac{2500 \times (16.67)^2}{60} = 11578.7 \text{ N}$$

$$C_c = F_c \cdot h = 11578.7 \times 0.6 = 6947.2 \text{ Nm}$$

The reactions due to this couple are vertically downwards on the inner wheels and vertically upwards on the outer wheels. The magnitude of this reaction on each of the inner and outer wheels,

$$\frac{Q}{2} = \frac{C_c}{2a} = \frac{6947.2}{3} = 2315.7 \text{ N}$$

$$\text{Load on the inner front wheel} = \frac{W_1}{2} - \frac{P}{2} - \frac{F}{2} - \frac{Q}{2}$$

$$= 7357.5 - 12.35 - 7.01 - 2315.7 = 5022.44 \text{ N}$$

$$\text{Load on the front outer wheel} = \frac{W_1}{2} + \frac{P}{2} - \frac{F}{2} + \frac{Q}{2}$$

$$= 7357.5 + 12.35 - 7.01 + 2315.7 = 9678.54 \text{ N}$$

$$\text{Load on the rear inner wheel} = \frac{W_2}{2} - \frac{P}{2} + \frac{F}{2} - \frac{Q}{2}$$

$$= 4905 - 12.35 + 7.01 - 2315.7 = 25814.96 \text{ N}$$

$$\text{Load on the rear outer wheel} = \frac{W_2}{2} + \frac{P}{2} + \frac{F}{2} + \frac{Q}{2}$$

$$= 4905 + 12.35 + 7.01 + 2315.7 = 7240.06 \text{ N}$$

Example 14.12

A four-wheel trolley car of total mass 2500 kg running on rails of 1.6 m gauge, negotiates a curve of 40 m radius at 60 km/h. The track is banked at 10°. The wheels have an external diameter of 0.8 m and each pair with axle has a mass of 250 kg. The radius of gyration for each pair is 0.4 m. The height of centre of gravity of the car above the wheel base is 0.9 m. Determine the pressure on each rail, allowing for centrifugal and gyroscopic couple actions.

■ Solution

Here $m = 2500$ kg, $a = 1.6$ m, $R = 40$ m, $v = \frac{60 \times 1000}{3600} = 16.67$ m/s, $\theta = 10°$, $r_w = 0.4$ m, $m_w = 250$ kg, $K_w = 0.4$ m, $h = 0.9$ m.

Let R_A and R_B be the reactions at A and B respectively. The various forces acting on the trolley car are shown in Fig.14.13.

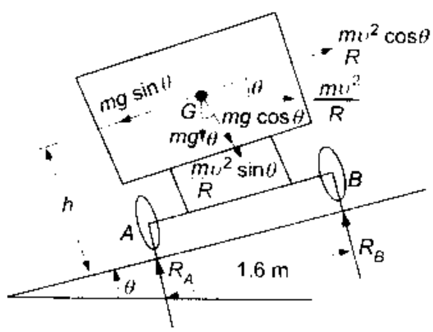


Fig.14.13 Force diagram

Resolving the forces perpendicular to the track, we have

$$R_A + R_B = mg\cos\theta + \frac{mv^2\sin\theta}{R}$$

$$= 2500 \times 9.81 \times \cos 10° + \frac{2500 \times (16.67)^2 \times \sin 10°}{40}$$

$$= 24152.4 + 3015.93$$

$$= 27168.33 \text{ N}$$

Now taking moments about B, we have

$$R_A \times a = \left(mg\cos\theta + \frac{mv^2\sin\theta}{R}\right)\cdot\left(\frac{a}{2}\right) + \left(mg\sin\theta - \frac{mv^2\cos\theta}{R}\right)\cdot h$$

$$R_A = \left[2500 \times 9.81 \times \cos 10° + \frac{2500 \times (16.67)^2 \times \sin 10°}{40}\right] \times 0.5$$

$$+\left[2500 \times 9.81 \times \sin 10° - \frac{2500 \times (16.67)^2 \times \cos 10°}{40}\right]\cdot (0.9/1$$

$$= 13584.17 - 7225.58 = 6358.59 \text{ N}$$

$$R_B = 20809.74 \text{ N}$$

$$\omega_w = \frac{v}{r_w} = \frac{16.67}{0.4} = 41.675 \text{ rad/s}$$

$$\omega_p = \frac{v}{R} = \frac{16.67}{40} = 0.41675 \text{ rad/s}$$

$$C = I\cdot\omega\cos\theta\cdot\omega_p$$

$$= m_w K_w^2 \cdot \omega_w \cos\theta \cdot \omega_p$$

$$= 250 \times (0.4)^2 \times 41.675 \times cos\, 10° \times 0.41675$$

$$= 684.17 \text{ Nm}$$

The force at each pair of wheels on each rail due to the gyroscopic couple,

$$P = \frac{C}{a}$$

$$= \frac{684.17}{1.6} = 427.6 \text{ N}$$

Due to this force the car would tend to overturn about the outer wheels. Total pressure on the inner rail,

$$P_i = R_A - P$$

$$= 6358.59 - 427.6$$

$$= 5930.99 \text{ N}$$

Pressure on the outer rail,

$$P_o = R_B + P$$

$$= 20809.74 + 427.6$$

$$= 21237.34 \text{ N}$$

Example 14.13

A gyrowheel *D* of mass 0.6 kg and radius of gyration 20 mm is mounted in a pivoted frame *C* as shown in Fig.14.14. The axis *AB* of the pivots passes through the centre of rotation *O* of the wheel, but the centre of gravity *G* of the frame *C* is 10 mm below *O*. The frame has a mass of 0.25 kg and the speed of rotation of the wheel is 3000 rpm in the counter-clockwise direction.

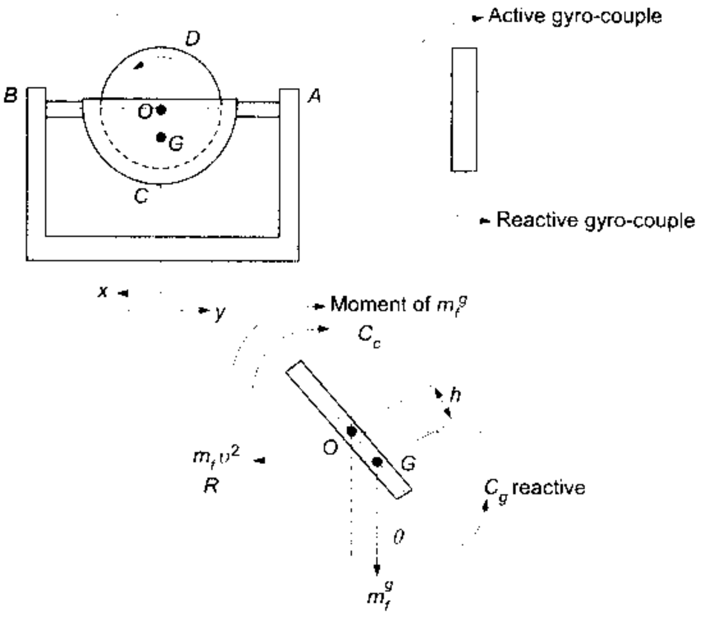


Fig.14.14 Gyrowheel mounted in a pivoted frame

The entire unit is mounted on a vehicle so that the axis *AB* is parallel to the direction of motion of the vehicle. If the vehicle travels at 15 m/s in a curve of 60 m radius, find the inclination of the gyrowheel from the vertical, when (a) the vehicle moves in the direction of the arrow *X* taking a left hand turn along the curve and (b) the vehicle reverses at the same speed in the direction of arrow *Y* along the same path.

Solution

Here $m_w = 0.6$ kg, $K_w = 0.02$ m, $OG = h = 0.01$ m, $m_f = 0.25$ kg, $N = 3000$ rpm,

$$v = 15 \text{ m/s}, \; R = 60 \text{ m}.$$

$$\omega = \frac{2\pi \times 3000}{60} = 314.2 \text{ rad/s}$$

$$I_w = m_w K_w^2 = 0.6 \times (0.02)^2 = 0.00024 \text{ kgm}^2$$

$$\omega_p = \frac{v}{R} = \frac{15}{60} = 0.25 \text{ rad/s}$$

Let θ = angle of inclination of gyrowheel from the vertical.

(a) Vehicle moving in the direction of arrow *X* while taking left turn along the curve.

Gyro-couple about *O*.

$$C_g = I_w \cdot \omega \cdot \omega_p \cos\theta = 0.00024 \times 314.2 \times 0.25 \times \cos\theta$$

$$= 0.018852 \cos\theta \text{ Nm}$$

Centrifugal couple about O, $C_c = \left(\frac{m_f v^2}{R}\right) \cdot h\cos\theta = \left(\frac{0.25 \times 15^2}{60}\right) \times 0.01 \times \cos\theta$

$= 0.009375\cos\theta$ Nm

Total overturning couple, $C_o = C_g - C_c = (0.018852 - 0.009375)\cos\theta$

$= 0.009477\cos\theta$ Nm (ccw)

Balancing couple due to the weight of the frame,

$$C_b = m_f gh\sin\theta$$
$$= 0.25 \times 9.81 \times 0.01 \times \sin\theta$$
$$= 0.024525\sin\theta \text{ Nm (cw)}$$

For the equilibrium condition,

$$C_o = C_b$$
$$0.009477\cos\theta = 0.024525\sin\theta$$
$$\tan\theta = 0.38642$$
$$\theta = 21.12°$$

(b) Vehicle reverses at the same speed in the direction of arrow Y along the same path.

$$C_o = C_g + C_c = (0.018852 + 0.009375)\cos\theta$$
$$= 0.028227\cos\theta$$

For the equilibrium condition,

$$C_o = C_b$$
$$0.028227\cos\theta = 0.024525\sin\theta$$
$$\tan\theta = 1.15095$$
$$\theta = 49.01°$$

14.10 GYROSCOPIC ANALYSIS OF GRINDING MILL

A grinding mill uses the gyroscopic effects to boost the crushing force. The grinding mill is shown in Fig.14.15. It consists of a conical roller which is placed symmetrically in a pan and is free to rotate on a shaft which is hinged to the central driving shaft. When the driving shaft rotates, the roller moves around the pan and crushes the material placed within it. The crushing is caused not only by the weight of the roller but by additional force which is produced by gyroscopic action.

Let ω = angular velocity of roller, represented by OA

ω_1 = angular velocity of driving shaft, represented by OB

ω_r = resultant velocity vector

Point D is the intersection of the vector for ω_r and the line of contact between roller and pan floor. At this point, the roller will have no relative velocity with respect to the pan floor.

In ΔOAC, we have

$$\frac{OA}{\sin(\theta - \phi)} = \frac{AC}{\sin\phi}$$

or

$$\frac{OA}{AC} = \frac{\sin(\theta - \phi)}{\sin\phi} = \frac{\omega}{\omega_1}$$

Also at point D,

$$\frac{\omega}{\omega_1} = \frac{r_o}{r}$$

where r = radius of the roller at the cross-section containing point D.

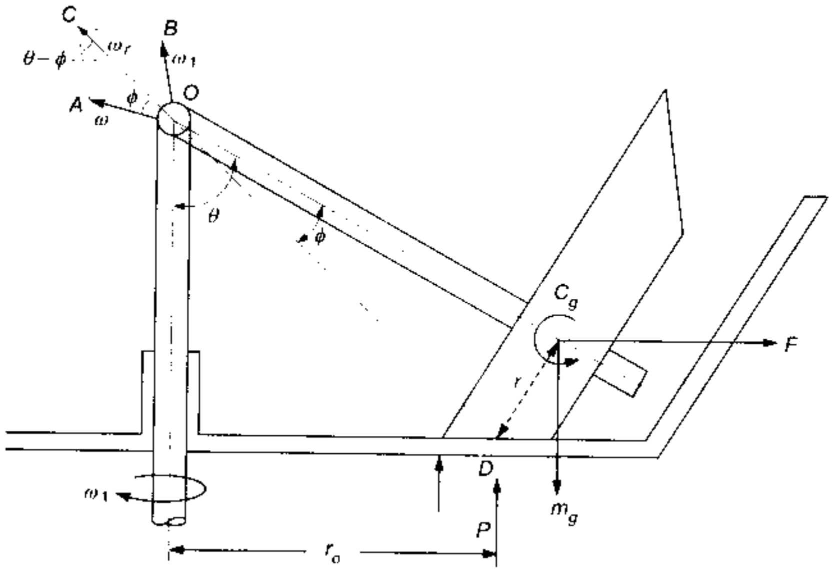


Fig.14.15 Grinding mill

The gyroscopic couple,

$$C_g = I_p\omega\omega_1\sin\theta + (I_p - i_e)\omega_1^2\sin\theta\cos\theta$$

$$= I_p\omega_1^2\left(\frac{r_o}{r}\right)\sin\theta + (I_p - I_e)\omega_1^2\sin\theta\cos\theta$$

$$= \left[(I_p - I_e)\cos\theta + \frac{I_p r_o}{r}\right]\omega_1^2\sin\theta \qquad \text{(a)}$$

By taking moments of all forces, we have

$$C_g = F(r_o + r\cos\theta)\cot\theta - mg(r_o + r\cos\theta) + Pr_o \qquad \text{(b)}$$

where P = total crushing force

m = mass of roller

F = centrifugal force

From (a) and (b), we get

$$\frac{P}{mg} = \left[1 + \left(\frac{r}{r_o}\right)\cos\theta\right] + \frac{\omega_1^2\sin\theta}{mgr_o}\left[(I_p - I_e)\cos\theta + I_p\left(\frac{r_o}{r}\right)\right] - \frac{F\cot\theta}{mg}\left[1 + \left(\frac{r}{r_o}\right)\cos\theta\right] \qquad (14.19)$$

For $\theta = 90°$, that is when the roller is circular, we get

$$\frac{P}{mg} = 1 + \frac{I_p\omega_1^2}{mgr}$$

or

$$P = mg + \frac{I_p\omega_1^2}{r} \qquad (14.20)$$

Example 14.14

In a crushing mill for cereals, the mass of roller is 100 kg. The roller is cylindrical in shape with 100 cm diameter. The polar mass moment of inertia of each roller is 160 kgm^2. The driving shaft runs at 90 rpm and the radius of roller at the centre of the grinding point is 70 cm. Determine the total crushing force.

■ **Solution**

Here $\theta = 90°$, $r = \frac{100}{2} = 50$ cm $= 0.5$ m, $m = 100$ kg, $I_p = 16$ kgm^2, $r_a = 0.7$ m

$$\omega_1 = \frac{2\pi \times 90}{60} = 9.42 \text{ rad/s}$$

$$\frac{P}{mg} = 1 + \frac{I_p \omega_1^2}{mgr}$$

$$= 1 + \frac{16 \times (9.42)^2}{(100 \times 9.81 \times 0.5)} = 3.89$$

Exercises

1 An aeroplane makes a complete half circle of 60 m radius towards left when flying at 250 km/h. The rotary engine and the propeller of the plane has a mass of 450 kg with a radius of gyration of 300 mm. The engine runs at 2400 rpm clockwise when viewed from the rear. Find the gyroscopic effect on the aircraft.

2 The rotor of the turbine of a ship makes 1500 rpm clockwise when viewed from the stern. The rotor has a mass of 800 kg and its radius of gyration is 300 mm. Find the maximum gyro-couple transmitted to the hull when the ship pitches with maximum angular velocity of 1 rad/s. What is the effect of this couple?

3 The mass of a turbine rotor of a ship is 8000 kg and has a radius of gyration of 0.75 m. It rotates at 1800 rpm clockwise when viewed from the stern. Determine the gyroscopic effects in the following cases:

(a) If the ship traveling at 100 km/h steers to the left along a curve of 80 m radius.

(b) If the ship is pitching and the bow is descending with maximum velocity. The pitching is with simple harmonic motion with periodic time of 20 s and the total angular movement between extreme positions is 10°.

(c) If the ship is rolling with an angular velocity of 0.03 rad/s clockwise when looking from stern.

In each case, determine the direction in which the ship tends to move.

4 A rail car has a total mass of 4000 kg. The moment of inertia of each wheel together with its gearing is 20 kgm^2. The centre distance between the two wheels on an axle is 1.5 m and each wheel is 400 mm radius. Each axle is driven by a motor, the speed ratio between the two being 1:14. Each motor with its gear has a moment of inertia of 15 kgm^2 and runs in a direction opposite to that of its axle. The centre of gravity of the car is 1 m above the rails.

Determine the limiting speed for the car when moving on a curve of 250 m radius such that no wheel leaves the rails.

5 A four wheel trolley car of total mass 2000 kg running on rails of 1 m gauge, rounds a curve of 30 m radius at 45 km/h. The track is banked at 10°. The wheels have an external diameter of 0.6 m and each pair of an axle has a mass of 250 kg. The radius of gyration of each pair is 250 mm. The height of the centre of gravity of the car above the wheel base is 1 m. Allowing for centrifugal force and gyroscopic couple action, determine the pressure on each rail.

6 A disc has a mass of 25 kg and a radius of gyration about its axis of symmetry 120 mm while its radius of gyration about a diameter of the disc at right angles to the axis of symmetry is 80 mm. The disc is pressed on to the shaft but due to incorrect boring, the angle between the axis of symmetry and the actual axis of rotation is 0.3°, though both these axes pass through the centre of gravity of the disc. Assuming that the shaft is rigid and is carried between bearings 200 mm apart, determine the bearing forces due to the misalignment at a speed of 4800 rpm.

7 The moment of inertia of each wheel of a motor cycle is 1.5 kgm^2. The rotating parts of the engine of the motor cycle have a moment of inertia of 0.28 kgm^2. The speed of the engine is six times the speed of the wheels and is in the same direction. The mass of the motor cycle is 250 kg and its centre of gravity is 0.6 m above the ground level.

Find the angle of heel if the motor cycle is travelling at 45 km/h and is taking a turn of 30 m radius. The wheel diameter is 0.6 m .

8 A racing motor cycle travels at 150 km/h round a curve of 120 m radius measured horizontally. The motor cycle and rider have a mass of 160 kg and their centre of gravity lies at 0.75 m above the ground level when the motor cycle is vertical. Each wheel is 0.6 m in diameter and has moment of inertia about its axis of rotation 1.5 kgm^2. The engine has rotating parts whose moment of inertia about their axis of rotation is 0.3 kgm^2 and rotates at five times the wheel speed in the same direction.

Find (a) the correct angle of banking of the track so that there is no tendency to side slip and (b) the correct angle of inclination of the motor cycle and the rider to the vertical.

9 A diesel locomotive moving at a speed of 100 km/h turns around a curve of radius 400 m to the right. The pair of driving wheels are 2 m in diameter and the mass along with the axis is 2000 kg. The radius of gyration of the wheels together with the axle may be taken as 0.6 m. Find the gyro effect on the pair of driving wheels.

10 A small high speed ship is driven by a turbine, the rotor of which is rotating at 12000 rpm in a clockwise direction, when viewed from the bow. The moment of inertia of the rotor is 15 kgm^2 and the ship is travelling at 20 m/s in a curve of 600 m radius, the direction being clockwise when viewed from the above.

Determine the gyroscopic couple acting on the ship and its effect on the ship.

11 The propeller shaft of an aero-engine is rotating at 1800 rpm. If the distance between the two bearings of the propeller shaft is 1 m and radius of gyration of propeller is 0.75 m, find the extra pressure on the bearings, when the aeroplane is whirling round in a horizontal circle of 300 m radius at a speed of 300 km/h. The mass of the propeller is 60 kg.

12 The turbine rotor of a ship which rotates clockwise when viewed from aft has a mass of 1500 kg, radius of gyration 0.7 m, and a speed of 2400 rpm. The ship pitches 5° above and below the horizontal position with simple harmonic motion of period 24 s. Determine the maximum reaction couple exerted by the rotor on the ship and the direction in which the bow will turn when falling.

13 A turbine rotor of a ship is of 2000 kg mass and has a radius of gyration of 0.8 m. Its speed is 2000 rpm. The ship pitches 5° above and below the mean position. A complete oscillation takes place in 20 s and the motion is simple harmonic. Determine

(a) the maximum couple tending to shear the holding down bolts of the turbine,
(b) the maximum acceleration of the ship during pitching and
(c) the direction in which the bow will tend to turn while rising, if the rotation of the rotor is clockwise, when looking from aft.

14 The rolling moment on a ship at a given instant is 12×10^6 Nm clockwise when viewed from the rear. The rotor of the stabilizing gyroscope is of 12×10^4 kg mass and spins at 1200 rpm clockwise when viewed from above. If the radius of the wheels about the spin axis is 2 m, determine the angular velocity of the precession to maintain the ship in an upright position.

15 A racing car of mass 2000 kg has a wheel base of 2 m and track width of 1 m. The centre of gravity lies midway between the front and rear axles and is 0.4 m above the ground. The engine of the car has a flywheel rotating in a clockwise direction when seen from the front at 6000 rpm. The moment of inertia of the flywheel is 50 kgm^2. if the car takes a curve of 15 m radius towards right, while running at 45 km/h, find the reaction between the wheels and the ground considering the gyroscopic and centrifugal effects of the flywheel and the weight of the car respectively.

16 For a single cylinder engine determine the bearing forces caused by the gyroscopic action of the flywheel (I = 0.32 kgm^2) as the engine traverses a 305 m radius curve at 96.6 km/h in a turn to the right. The engine speed is 3300 rpm and it is turning clockwise when viewed from the front of the engine. The centre distance between the bearings is 152 mm.

17 Explain the following:

(a) Gyroscopic stabilization of sea vessels

(b) Effect of gyroscopic couple on the stability of an automobile negotiating a curve

(c) What are the principle of a gyroscope? Discuss the factors that effect the stability of an automobile while negotiating a curve.

(d) How is the magnitude and direction of the Gyroscopic couple fixed?

(e) Describe the effect of the Gyroscopic couple on pitching, rolling and steering of a ship with neat sketches indicating the direction of couple vector, spin vector and precession vector.

(f) Deduce an expression for the couple that is called into play in the case of a wheel rotating with uniform angular velocity in order to maintain a given rate of precession.

18 A thin circular disc is fitted to a shaft as shown in Fig.14.16. Weight of the disc is 500 N and diameter is 1.2 m. Shaft rotates at 300 rpm in the counter-clockwise direction when seen from the right side. Find the effect of gyroscopic couple on the shaft and the bearing reactions at A and B taking the effect of weight of the disc.

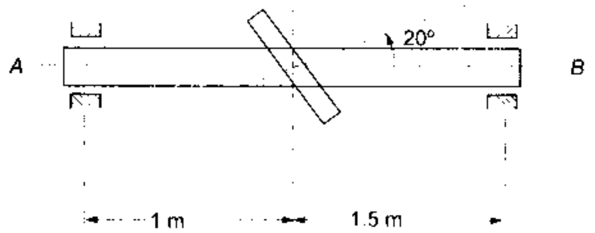


Fig.14.16 Thin circular disc on a shaft

19 Explain what you understand by gyroscopic stabilization. Illustrate with the help of a sketch how this is carried out in ships. Obtain a relation between the gyroscopic torque and the couple applied by the waves for complete stabilization if the waves be sinusoidal.

20 The rotor of a turbojet engine has a mass of 200 kg and a radius of gyration 250 mm. The engine rotates at a speed of 10000 rpm in the clockwise direction if viewed from the front of the aeroplane. The aeroplane while flying at 1000 km/h turns with a radius of 2 km to the right. Compute the gyroscopic moment exerted by the rotor on the plane structure. Also determine whether the nose of the plane tends to rise or fall when the plane turns.

21 The moment of inertia of each wheel of a motor cycle is 2 kgm^2. The rotating parts of the engine of the cycle have a moment of inertia of 0.3 kgm^2. The speed of the engine is 6 times the speed of the wheels and in the same sense. The weight of the motor cycle together with the rider is 2600 N and its centre of gravity is 0.6 m above the ground level when the cycle is standing upright and rider is sitting on it. Find the average angle of inclination with vertical for equilibrium if the cycle is travelling at 60 km/h and taking a turn of 30 m radius. The wheel diameter is 0.6 m.

22 A disc of mass 100 kg and radius of gyration 0.5 m is supported as shown in Fig.14.17. When the disc is rotating at 10 rad/s the cord on the right hand side bearing gets broken. Discuss the motion of the disc.

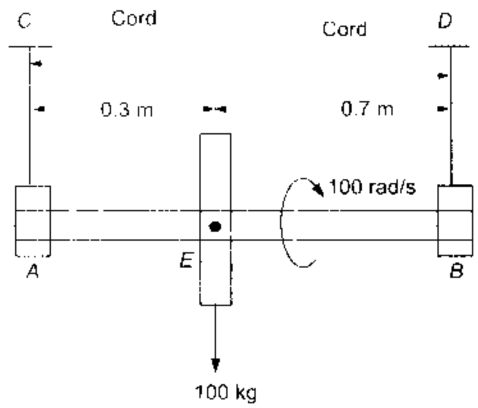


Fig.14.17 Gyration of disc

23 A uniform disc of 250 mm diameter has a mass of 15 kg. It is mounted centrally on a horizontal shaft running in bearings 200 mm apart. The disc spins with a uniform speed of 1800 rpm in vertical plane in the counter-clockwise direction looking from right hand side bearing. The shaft precesses with a uniform speed of 60 rpm in horizontal plane in counter-clockwise direction when looking from top. Determine the bearing reactions due to the disc mass and gyroscopic effects.

24 A racing car of mass 3000 kg has a wheel base of 2.5 m and track of 1.5 m. The centre of gravity is located 0.6 m above the ground level and 1.5 m from the rear axle. Each wheel is of 1 m diameter and 0.8 kgm^2 moment of inertia. The back axle ratio is 4.5. The drive shaft engine flywheel and transmission are rotating clockwise when viewed from the front with equivalent mass of 150 kg with radius of gyration 0.2 m. Determine the load distribution on the wheels if the car is rounding a curve of 80 m radius at 120 km/h when (a) taking a right turn and (b) taking a left turn.